THE ESTIMATION OF HIGH ORDER GENERALIZED MODULUS OF CONTINUITY IN $L^p_w$ 

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In this paper we deal with the estimation of the best approximation and generalized modulus of continuity of derivatives of periodic functions in weighted reflexive Lebesgue spaces. In unweighted Lebesgue spaces the inequalities for classical modulus of continuity and best approximations of derivatives were derived in the papers [1], [2].

1. Some definitions

Let $T$ denote the interval $(-\pi, \pi)$. A positive almost everywhere, integrable function $w : T \rightarrow (0, \infty)$ is called a weight function. With any given weight $w$ we associate the $w$-weighted Lebesgue space $L^p_w(T)$ consisting of all measurable functions $f$ on $T$ such that

$$\|f\|_{L^p_w(T)} = \|fw\|_{L^p(T)} < \infty.$$ 

Let $1 < p < \infty$ and $1/p + 1/q = 1$. A weight function $w$ belongs to the Muckenhoupt class $A_p(T)$ if

$$\left( \frac{1}{|I|} \int_I w^p(x) \, dx \right)^{1/p} \left( \frac{1}{|I|} \int_I w^{-q}(x) \, dx \right)^{1/q} \leq C$$

with a finite constant $C$ independent of $I$, where $I$ is any subinterval of $T$ and $|I|$ denotes the length of $I$.

Let $1 < p < \infty$ and $w \in A_p(T)$. We define an operator on $L^p_w(T)$ by

$$\sigma_h(g)(x) = \frac{1}{2h} \int_{x-h}^{x+h} g(t) \, dt, \quad 0 < h < \pi$$

It is known that the operator $\sigma_h$ is bounded uniformly with respect to $h$ in $L^p_w(T)$, when $w \in A_p(T)$, $1 < p < \infty$. The modulus of continuity $\Omega_h g$ of $g \in L^p_w(T)$ is defined by

$$\Omega_h g \in L^p_w = \sup_{0 < \delta < h} \| \frac{1}{h} \sum_{1 \leq \ell \leq \delta} (I - a_{\ell h}) g \|_{L^p_w}.$$ 

The best approximation of $f \in L^p_w(T)$ in the class $\Pi_n$ of trigonometric polynomials of degree not exceeding $n$ is defined by

$$E_n(f)_{L^p_w} = \inf \left\{ \| f - T_n \|_{L^p_w} : T_n \in \Pi_n \right\}.$$ 

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