SIMULTANEOUS AND CONVERSE APPROXIMATION THEOREMS IN WEIGHTED LEBESGUE SPACES

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Abstract. In this paper we deal with the simultaneous and converse approximation by trigonometric polynomials of the functions in the Lebesgue spaces with weights satisfying so called Muckenhoupt’s $A_p$ condition.

1. Introduction and the main results

Let $T := [-\pi, \pi]$. A positive almost everywhere (a.e.), integrable function $w : T \to [0, \infty]$ is called as a weight function. With any given weight $w$ we associate the $w$-weighted Lebesgue space $L^p_w(T)$ consisting of all measurable functions $f$ on $T$ such that

$$
\|f\|_{L^p_w(T)} = \|f w\|_{L^p(T)} < \infty.
$$

Let $1 < p < \infty$ and $1/p + 1/q = 1$. A weight function $w$ belongs to the Muckenhoupt class $A_p(T)$ if

$$
\left( \frac{1}{|I|} \int_I w^p(x)dx \right)^{1/p} \left( \frac{1}{|I|} \int_I w^{-q}(x)dx \right)^{1/q} \leq c
$$

with a finite constant $c$ independent of $I$, where $I$ is any subinterval of $T$ and $|I|$ denotes the length of $I$.

For formulation of the new results we will begin with some required informations. Let

$$
f(x) \sim \sum_{k=-\infty}^{\infty} c_k e^{ikx} = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)
$$

and

$$
\tilde{f}(x) \sim \sum_{k=1}^{\infty} (a_k \sin kx - b_k \cos kx)
$$


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