ON THE INVERSE INEQUALITIES FOR TRIGONOMETRIC POLYNOMIAL APPROXIMATIONS IN WEIGHTED LORENTZ SPACES

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1. Introduction

The Weierstrass well-known theorem on the approximation of the continuous function by the trigonometric polynomials and its quantitative refinement represented by Jackson inequality

\[ E_n(f) \leq C \omega(f, \frac{1}{n}) \]  

(1.1)

are one of the basement of the Approximation Theory.

In the inequality (1.1), for \(2\pi\)-periodic continuous function \(f\), \(E_n(f)\) denotes the best approximation of \(f\) by the trigonometric polynomials, i.e.

\[ E_n(f) = \inf_{k \in [0,2\pi]} \max_{x \in [0,2\pi]} |f(x) - T_k(x)|, \]

where the infimum is taken over all trigonometric polynomials of order \(k \leq n\), and

\[ \omega(f, \delta) = \sup_{|h| \leq \delta} \max_{x \in [0,2\pi]} |f(x + h) - f(x)| \]

denotes the modulus of continuity of \(f\). The analog of Jackson inequality is correct for the mean approximation and higher order modulus of continuity as well (see [1]).

Yet by 1913, S. Bernstein [2], obtained the reversed estimations of Jackson’s inequality in the space of continuous functions for some specific cases. Later Quade [3], brothers A. and M. Timan [4], S. B. Stechkin [5], M. Timan [6], etc. proved the reversed type inequalities of Jackson’s inequality, including in \(L^p\), \(1 < p < \infty\), spaces. These type inequalities played an important role in the investigation of properties of the conjugate functions [7], in the study of absolute convergent Fourier series [8], and in the related problems. In the weighted Lebesgue spaces the inverse inequalities for classical module of smoothness and best approximations were derived in the papers [6], [8]. In [10] this result is extended for reflexive Orlicz space. For the approximation in weighted Lebesgue and Orlicz spaces we refer to [11], [12], [13], [14].

The order of generalized module of smoothness, as it has been shown in [6] and [9], depends not only on the rate of the best approximation but also on the metric of the spaces. In the present paper we reveal that the similar influence in weighted Lorentz spaces is expressed not only by the "leading" parameter of the space, but also by the second parameter in the definition of Lorentz spaces.

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