APPROXIMATION BY GENERALIZED FABER SERIES IN WEIGHTED BERGMAN SPACES ON INFINITE DOMAINS WITH A QUASICONFORMAL BOUNDARY

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ABSTRACT. Using an integral representation on infinite domains with a quasiconformal boundary the generalized Faber series for the functions in the weighted Bergman space $A^2(G, \omega)$ are defined and its approximation properties are investigated.

1. INTRODUCTION AND MAIN RESULTS

Let $G$ be a simply connected domain in the complex plane $\mathbb{C}$ and let $\omega$ be a weight function given on $G$. For functions $f$ analytic in $G$ we set

$$A^1(G) := \left\{ f : \int \int_G |f(z)| \, d\sigma_z < \infty \right\}$$

and

$$A^2(G, \omega) := \left\{ f : \int \int_G |f(z)|^2 \omega(z) \, d\sigma_z < \infty \right\},$$

where $d\sigma_z$ denotes the Lebesgue measure in the complex plane $\mathbb{C}$.

If $\omega = 1$, we denote $A^2(G) := A^2(G, 1)$. The space $A^2(G)$ is called the Bergman space on $G$. We refer to the spaces $A^2(G, \omega)$ as “weighted Bergman spaces”. It becomes a normed spaces if we define

$$\|f\|_{A^2(G, \omega)} := \left( \int \int_G |f(z)|^2 \omega(z) \, d\sigma_z \right)^{1/2}.$$

Now, let $L$ be a finite quasiconformal curve in the complex plane $\mathbb{C}$. We recall that $L$ is called a quasiconformal curve if there exists a quasiconformal homeomorphism of the complex plane onto itself that maps a circle onto $L$. We denote by $G_1$ and $G_2$ the bounded and unbounded components of $\mathbb{C} \setminus L$

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