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IMPACT OF VISCOUS DAMPING MODELS ON NONLINEAR RESPONSE OF SDOF SYSTEMS

U. Hasgul[1] and M. J. Kowalsky[2]

ABSTRACT

Within the context of Performance-Based Seismic Design, accurate prediction of non-linear response is essential to control structural performance. While variables such as strength, hysteretic response and earthquake characteristics impact non-linear response, the effect of the choice of viscous damping model on non-linear response using different hysteretic models in frame analysis is less obvious. Analysts will often utilize the default models available in commonly available analysis codes, which is typically proportional to the damping force considering the initial stiffness. In the research described in this paper, a parametric study was conducted using over 100 real earthquake ground motions and five different hysteretic models to assess the impact of the choice of viscous damping model on the non-linear response of single degree of freedom systems. The results of the analyses indicate that while there is variation, the choice of damping model can have a profound impact on inelastic analysis, with differences of 50% in terms of peak displacements common. Increased levels of ductility tend to have a more pronounced effect, while choice of hysteretic model also plays a role with larger differences apparent for hysteretic models with lower levels of residual displacements.

[1]Post-Doctoral Researcher, Dept. of Civil Engineering, Balikesir University, TURKEY.
[2]Professor, Dept. of Civil, Construction and Environmental Eng., North Carolina State University, Raleigh, USA.

Correspondence to: Mervyn J. Kowalsky; Department of Civil Engineering, North Carolina State University, Campus Box 7908, Raleigh / NC, USA, 27695. E-mail: kowalsky@ncsu.edu
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Impact of Viscous Damping Models on Nonlinear Response of SDOF Systems

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ABSTRACT

Within the context of Performance-Based Seismic Design, accurate prediction of non-linear response is essential to control structural performance. While variables such as strength, hysteretic response and earthquake characteristics impact non-linear response, the effect of the choice of viscous damping model on non-linear response using different hysteretic models in frame analysis is less obvious. Analysts will often utilize the default models available in commonly available analysis codes, which is typically proportional to the damping force considering the initial stiffness. In the research described in this paper, a parametric study was conducted using over 100 real earthquake ground motions and five different hysteretic models to assess the impact of the choice of viscous damping model on the non-linear response of single degree of freedom systems. The results of the analyses indicate that while there is variation, the choice of damping model can have a profound impact on inelastic analysis, with differences of 50% in terms of peak displacements common. Increased levels of ductility tend to have a more pronounced effect, while choice of hysteretic model also plays a role with larger differences apparent for hysteretic models with lower levels of residual displacements.

Introduction

It is common to specify a level of elastic damping in non-linear analysis to represent damping in initial stages of response. This is normally specified as a percentage (typically 2% to 5%) of critical damping. In addition, analysts can choose if the damping coefficient is proportional to initial or tangent stiffness, amongst other options. Typically, research papers reporting results on Single-Degree-Of-Freedom (SDOF) state that the 5% elastic damping was used without clarifying whether this has been related to the initial or tangent stiffness. With multi-degree-of-freedom (MDOF) analyses, the situation is often further confused by the adoption of Rayleigh damping, which is a combination of mass-proportional and stiffness-proportional damping [1-3].

Many analysts consider the initial stiffness proportional damping to be rather insignificant for inelastic analyses of SDOF or MDOF systems. In the initial stiffness proportional approach, the damping coefficient is constant throughout the analysis, even in the inelastic range of response and is based on the initial elastic stiffness of system. On the other hand, the tangent stiffness approach uses the instantaneous value of the stiffness, hence the

[1]Post-Doctoral Researcher, Dept. of Civil Engineering, Balikesir University, TURKEY.
[2]Professor, Dept. of Civil, Construction and Environmental Eng., North Carolina State University, Raleigh, USA.

Correspondence to: Mervyn J. Kowalsky; Department of Civil Engineering, North Carolina State University, Campus Box 7908, Raleigh / NC, USA, 27695. E-mail: kowalsky@ncsu.edu
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damping coefficient is updated as the stiffness changes. For example, in case of use of *elasto-plastic* response, the tangent stiffness damping force will be zero while the structure deforms along a yield plateau [1]. Furthermore, the hysteretic models are generally calibrated to experimental structural response in the inelastic phase. Therefore additional elastic damping should not be used in the post-yield state to represent structural response except when the structure is unloading and reloading elastically. Research by others has shown that the impact of damping model, i.e., tangent stiffness vs. initial stiffness proportional, can have a large impact on the non-linear response of systems and as a consequence call into question well established displacement equivalence rules such as the equal energy and equal displacement approximation [2].

It is the goal of this paper to further study the observations made by [1, 2] with regards to the impact of choice of viscous damping model on inelastic displacement response. Results in [2], which were based on five artificial spectrum compatible EQ records and one real EQ record showed increased displacement when using tangent stiffness proportional damping. To further explore that outcome, a parametric study was conducted for the research in this paper using over 100 real earthquake ground motions and five different hysteretic models to assess the impact of the choice of viscous damping model on the non-linear response of SDOF systems. In order to achieve multiple levels of non-linear response, several SDOF oscillators with different heights, levels of axial load and moment strength were considered in order to investigate impact of the damping choice over a wide period range. In total, the research presents the outcome of over 45,000 non-linear time history analyses (NTHA).

Numerical Models and Study Parameters

Various SDOF oscillators, representing reinforced concrete cantilever bridge piers were considered in the non-linear analyses. The SDOF cantilever columns all have the same diameter but have different heights (See Fig. 1 $L / D = 4$, *7* and *10*), longitudinal steel ratios (See Fig. 2 ρ = *0.5%, 1.00%, 1.50%, 2.00% and 2.50%*) and axial load ($P = 0.05A_g f_c'$, $0.10A_g f_c'$ and $0.15A_g f_c'$) resulting in a wide range of initial periods between 0.40 to 3.30 sec. In total, 45 different SDOF oscillators were subjected to 100 different earthquake time histories using 5 different hysteretic models. All oscillators were analyzed using initial stiffness proportional damping (ICTYPE 0 in Ruaumoko [4]) and tangent stiffness proportional damping (ICTYPE 6 in Ruaumoko [4]), as shown in Figs. 1, and 2.

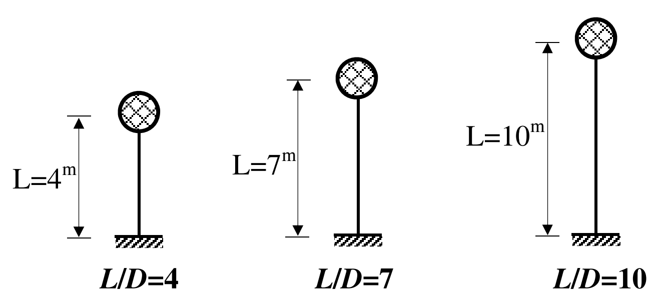

Figure 1. Aspect ratio of the considered SDOF oscillators

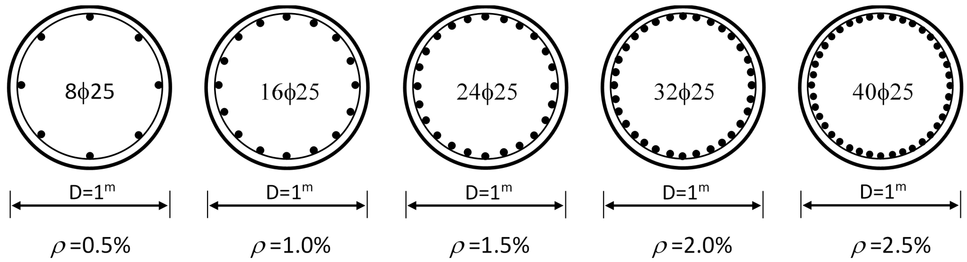

Figure 2. Number of the reinforcements and volumetric ratios for column members

Hysteretic Models

In the study, five hysteretic models, as shown in Fig. 3, were considered to investigate the effect of using the initial or tangent stiffness damping in the NTHA. The *Thin* (*Small*) *Takeda* and *Large Takeda* models represent reinforced concrete column and beam members, respectively. The *Ramberg-Osgood* model is appropriate for steel structures, while the *Ring-Spring* (*Flag-shaped*) resembles a post-tensioned column or wall. The *Bi-linear* shape was included because of its importance in seismic analysis, and also can represent various types of isolation systems [5].

Moment-curvature relationships for the RC column members were determined using Matlab code *CUMBIA* [6]. *CUMBIA* was developed for the design and analysis of RC members using unconfined and confined concrete models proposed by Mander *et al* [7,8] and the steel model proposed by King [9]. The parameters which define the smallest and largest loop area for the *Thin-Takeda* and *Large-Takeda* models were respectively selected as $\alpha=0.5$ $\beta=0$ and $\alpha=0$ $\beta=0.6$. The loop area parameter for the *Ring-Spring* was chosen as $\beta=0.35$. Note that this hysteretic shape is self-centering and has zero residual displacement, as shown in Fig. 3, which was obtained from the manual for the analysis code *Ruaumoko* [4].

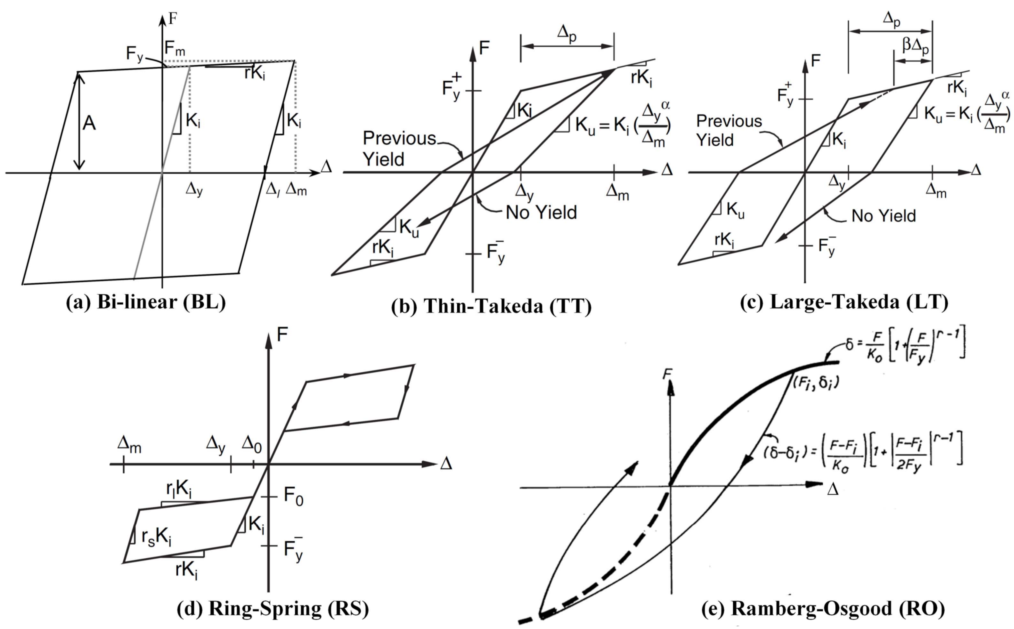

Figure 3. Hysteretic models considered in the study [4]

Earthquake ground motions

A large suite of real earthquake records were chosen to conduct this study, as opposed to artificial spectrum compatible records [2]. The suite of 100 records were from different site classes (B, C, D, E, and Near Fault) and were from earthquakes that had moment magnitudes ranging between 6.0 and 7.8 [10].

Non-linear Time History Analyses of SDOF Systems

In the analyses, the elastic damping which represents the damping in initial stages of cyclic response, is assumed as 5% of critical damping. The *P-delta* effects and moment-axial load interaction were not included in the non-linear analyses. The NTHA were carried out using the program RUAUMOKO, using *Newmark Constant Average Acceleration* integration with $\beta =$ 0.25 [4].

In order to show the difference between the responses of two damping approximations on the SDOF systems, the peak displacement ratios using the tangent stiffness damping model were compared to those obtained using the initial stiffness model for the each ground motion. The peak displacement ratios between the initial or tangent stiffness proportional damping model were computed by Eq. 1.

$$r_\delta = \frac{\delta_{TS}^{Peak}}{\delta_{IS}^{Peak}} \quad (1)$$

where the displacement ratio $\boldsymbol{\delta}$ obtained from the **T**angent **S**tiffness and **I**nitial **S**tiffness proportional damping is indicated by subscript "***TS***" and "***IS***", respectively (Fig. 4). As shown in Fig.4, when the displacement demand using the tangent stiffness damping is larger than for the initial stiffness damping, the corresponding displacement ratio is greater than 1.00. Results of the NTHA are presented in the next section to demonstrate the impact of choice of damping model. The displacement ratios (δ_{TS} / δ_{IS}) calculated by the peak displacement demands of SDOF oscillators are discussed in terms of the *period* (T), *displacement ductility* (μ_Δ) and *residual drift* ($\Delta^{Res.}$). Furthermore, the effect of axial load levels and soil types of the earthquake ground motions were evaluated using the related figures.

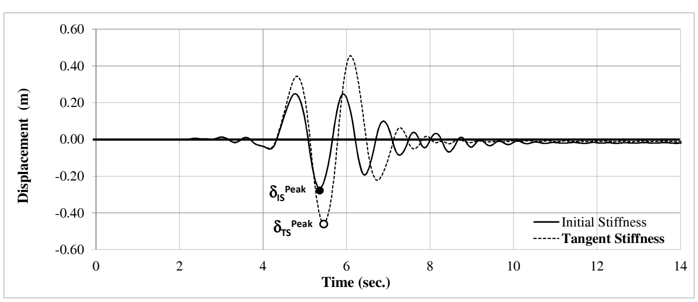

Figure 4. A sample displacement response using initial and tangent stiffness damping

Analysis Results and Impact of Damping Model Choice on Non-linear Response

After completing the NTHA of SDOF oscillators using the initial and tangent stiffness proportional damping models, the displacement ratios (δ_{TS} / δ_{IS}) corresponding to the peak displacement demands subjected to the ground motion records were plotted with respect to the parameters below.

Displacement Response with Initial Period

Fig. 5 presents the variations of displacement ratio δ_{TS} / δ_{IS} with period of the SDOF systems for all hysteretic models considered. Clearly there is a significant difference between the displacement response of the initial stiffness and tangent stiffness damping approaches. The peak displacement demands using tangent stiffness proportional damping is predominantly larger than those of the initial stiffness damping. As would be expected, the displacement ratios r_{δ} tend to decrease as the initial period of SDOF systems increases. But it should be noted that this trend is not valid for the *Ramberg-Osgood* model. The differences between both damping approaches are more critical particularly in the period range from ***0.4*** to ***0.85*** sec. which represents the short-period range of response, as shown in Fig.5. In order to show the extreme responses of the displacement ratios, the peak displacement ratios, in some cases, can be ***1.72***, ***2.43***, ***2.50***, ***4.01*** and ***1.72*** times for the *Bi-linear, Thin and Large-Takeda, Ring-Spring* and *Ramberg-Osgood* model, respectively, indicating significant, non-conservative influence when choosing initial stiffness proportional damping. The results of the NTHA indicate that the most unfavorable displacement ratio was observed for the *Ring-Spring* hysteretic model.

For a few column models that show fully elastic or slightly non-linear response, the displacement ratios can go below 1.00 (r_{δ}<1.00) for some ground motion records as shown from Fig.5. This means that the peak displacement demand of the structure using the initial stiffness damping is larger than for use of the tangent stiffness model. It is thought that the main reason of this response is due to the cyclic characteristics of hysteretic shape and the residual displacement remaining in the system. It should also be noted that this response is irrespective of the ground motion properties.

Displacement Response with Ductility Level

In determination of the displacement ductility μ_{Δ} of the SDOF cantilever systems, the response of the initial stiffness damping models were used. The yield displacement was calculated by means of the Eqs. 2 and 3 [5,11].

$$\delta_y = \phi_y \, (H+L_{sp})^2/3 \quad (2)$$

$$\phi_y = 2.25\varepsilon_y \,/\, D \quad (3)$$

where ***H*** is the column height, $\boldsymbol{L_{sp}}$ is the effective additional height representing strain penetration effects, $\boldsymbol{\varepsilon_y}$ is the yield strain of the flexural reinforcement and ***D*** is the section depth for the circular column.

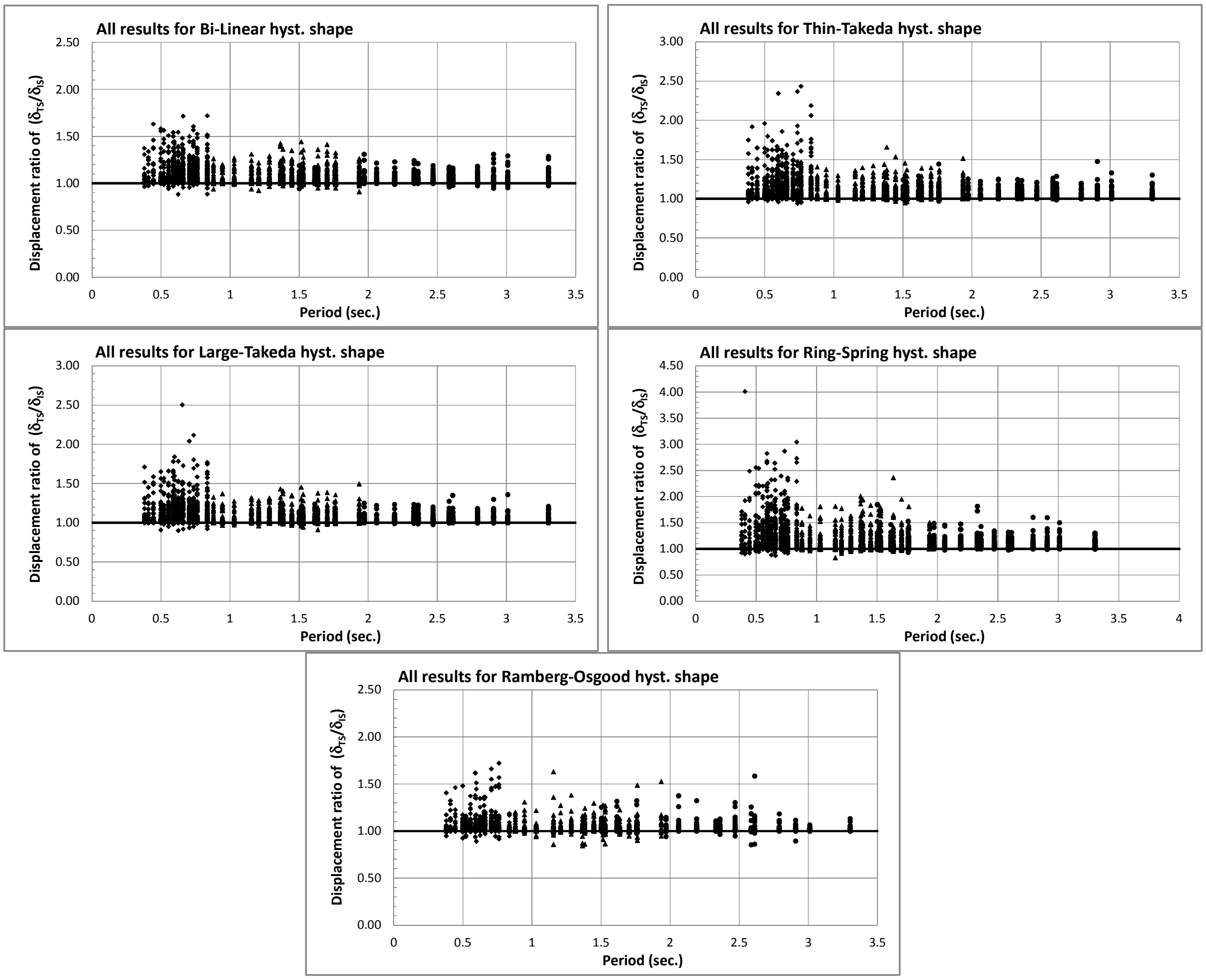

Figure 5. Variations of the displacement ratios (δ_{TS} / δ_{IS}) with period of SDOF oscillators

Fig. 6 presents the variations of displacement ratio with the ductility of the SDOF systems. As shown from Fig. 6, there is no difference on the displacement demands of the systems which respond fully elastic or slightly non-linear for ductility less than 2. It is apparent that the tangent stiffness damping models yields larger displacement when compared to the initial stiffness damping model as the ductility increases.

Effect of Residual Displacement on Displacement Response

As noted previously, the displacement ratios determined for some column models, which respond fully elastic or slightly non-linear during the ground motion records, can be lower than 1.00 for some ground motion records. The individual analysis results showed that the residual displacement remaining in the system can affect the displacement ratio. To further investigate this behavior, the variation of displacement ratio with residual drift, based on both the initial and tangent stiffness damping models ($\Delta_{IS}^{Res.} = \delta_{IS}^{Res.} / L$ and $\Delta_{TS}^{Res.} = \delta_{TS}^{Res.} / L$), are given in Fig.7 for all hysteretic models considered. It is interesting to note that lower residual drifts tend to give a wider gap between tangent and initial stiffness damping results. This is not surprising as large residual drifts can skew the peak response values dramatically as a system oscillates about an offset position.

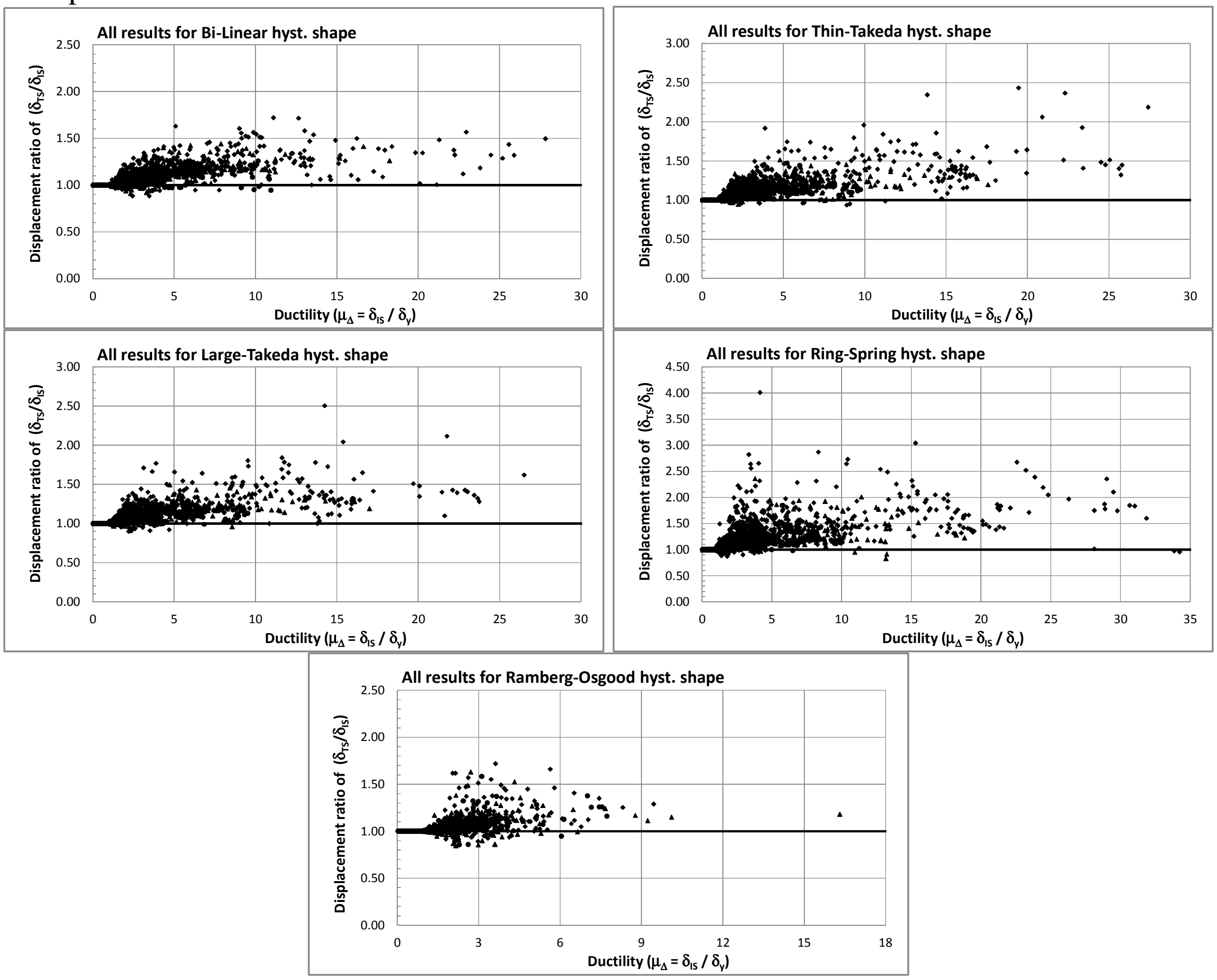

Figure 6. Variations of the displacement ratios (δ_{TS} / δ_{IS}) with ductility of SDOF oscillators

Effects of Axial Load and Site Classes of Ground Motions on Displacement Response

Impact of choice of the initial or tangent proportional damping in determining of displacement responses of the SDOF systems are also discussed in terms of the levels of axial load and site classes of the earthquake ground motions. The graphics corresponding to displacement ratio for both parameters are presented in Figs. 8, and 9 for only the *Bi-Linear* model for the sake of brevity. It should be said that all evaluations performed with regard to these parameters are also consistent for other hysteretic models considered in the study.

Referring to Fig. 8, it can be noted that changes in the level of axial load has no significant effect on the displacement response of the systems. The displacement ratios of SDOF systems which have nearly the same period range show similar response for different levels of axial load. Furthermore, as the site class of the ground motion records changes, the displacement ratios are significantly affected, as shown from Fig. 9. As would be expected, the displacement ratios determined from the *near-fault records* are dramatically larger than others since the largest ductility demands occurred during these analyses.

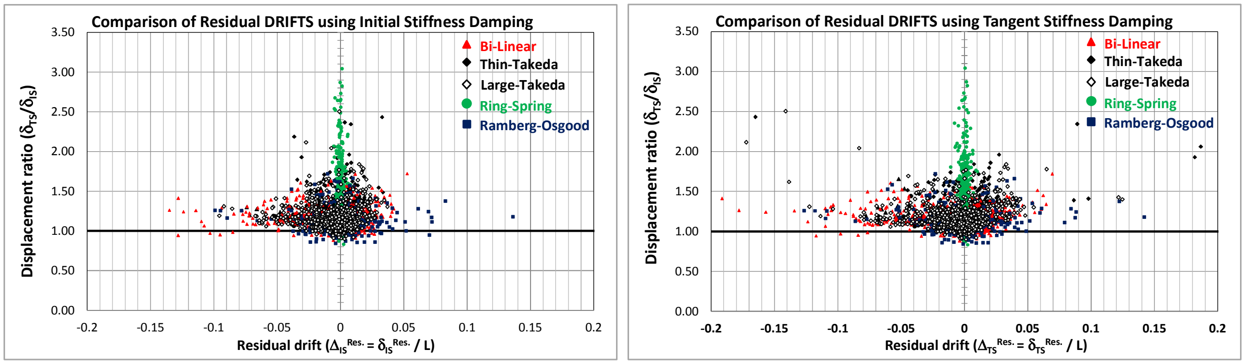

Figure 7. Comparison of displacement ratio δ_{TS}/δ_{IS} with residual drift relationships considering all hysteretic models

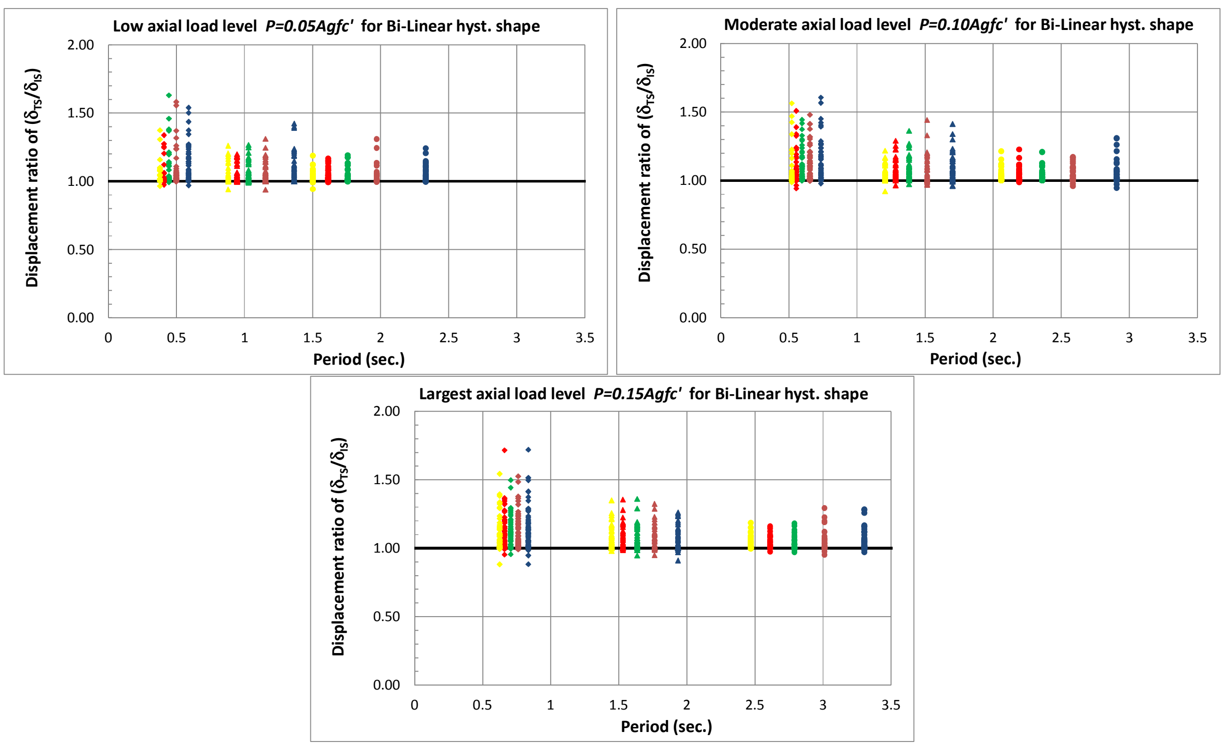

Figure 8. Effect of different axial load levels to displacement ratios (Bi-linear model)

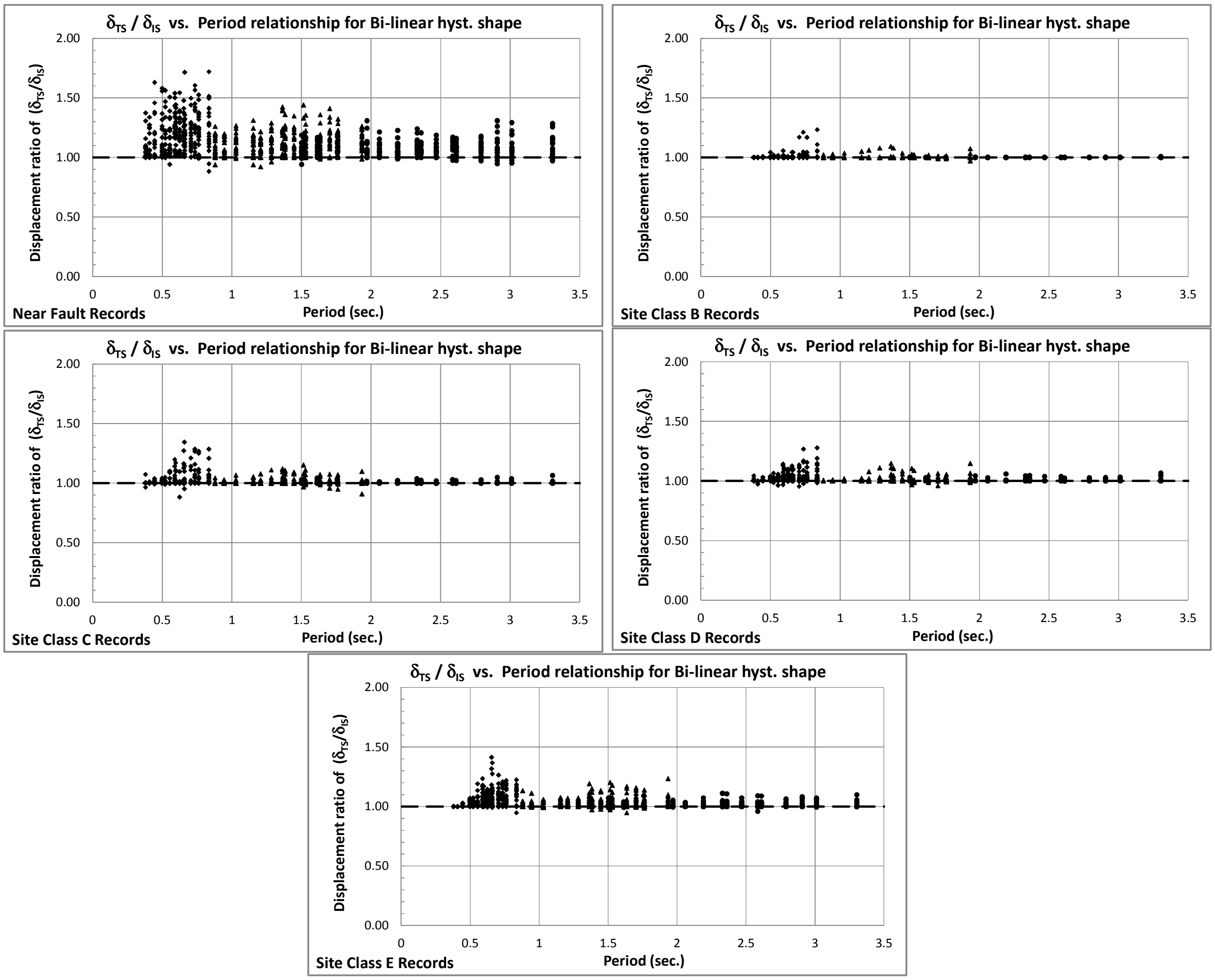

Figure 9. Effect of soil types of ground motion records to displacement ratios (Bi-linear model)

Conclusions

The impact of the choice of damping model is important, especially if the results of non-linear time history analysis indicate wide variations. This is of particular importance for assessing performance objectives within the context of *Performance-Based Seismic Design* where an accurate estimate of displacements of inelastic systems is important. Initial stiffness proportional damping results in damping forces which remain large after yielding, while tangent stiffness proportional damping results in large reductions in damping force [2]. Furthermore, past studies have shown that tangent stiffness proportional damping more accurately predicts actual structural response for yielding systems [1].

Presented in this paper are the results of NTHA of several SDOF oscillators subjected to a series of 100 ground motion records. Analyses were conducted using both initial and tangent stiffness proportional damping. The outcome of the analyses are consistent with prior studies [1], [2] which indicated that non-linear displacements of SDOF oscillators are generally much larger when tangent stiffness proportional damping is used. This is an important outcome as analysts often use initial stiffness proportional damping models due to limitations of the software used to conduct such analysis, and a lack of data showing the potential impact that the choice of damping model can have.

Although the analyses were conducted for only SDOF systems, it is thought that similar results would be obtained for MDOF systems. Further studies are underway to investigate this, as well comparisons to additional experimental shake table data, where available.

Acknowledgments

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