The 5th Asia Conference on Earthquake Engineering
October 16-18, 2014

IMPACT OF INITIAL AND TANGENT STIFFNESS PROPORTIONAL DAMPING MODELS ON DUCTILITY DEMANDS ON SDOF SYSTEMS

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ABSTRACT

This paper presents impact of viscous damping models on non-linear displacement response. In the research paper, a comprehensive parametric study has been conducted on various Single-Degree-of-Degree (SDOF) systems, in the period range of 0.40 sec. to 3.30 sec., using over 100 real earthquake ground motions and five different hysteretic models to assess the impact of the viscous damping model on the non-linear response of SDOF systems. In order to achieve multiple levels of non-linear response, oscillator moment strengths have been varied. In each case, the peak displacement demands were obtained from Non-linear Time History Analysis (NTHA), where either the initial or tangent stiffness damping was specified, and displacement ratios were calculated for the related ground motion records. In total, the research presents the outcome of over 45,000 NTHA. The results for all analyses are presented with respect to level of ductility, period, ground motion characteristics and residual displacements. Specific trends in individual analyses are explained as well.

The results of the analyses indicate that the choice of damping model is actually more impactful than the amount of damping assumed in analysis. The structural demands using the initial stiffness proportional damping generally underestimate the displacement and ductility demands when compared against the tangent stiffness proportional damping. The outcome of this research supports the recommendations made by others in the recent past that the tangent stiffness proportional damping should be used to predict non-linear response of systems.

Keywords: Initial/Tangent stiffness damping, Non-linear time history analysis, Earthquake engineering

INTRODUCTION

It is important to specify appropriate values of viscous damping models to determine structural response demands within context of *Performance Based Seismic Design*. This is normally specified as a percentage (typically 2% to 5%) of critical damping. While it has been common in the past to use initial stiffness proportional damping in Non-linear Time History Analysis (NTHA), recent research has shown that tangent stiffness proportional damping, where damping coefficient changes every time according to current stiffness, better represents the deformation demands of non-linear systems. With the case of *elasto-plastic* response, as the initial stiffness damping force is constant throughout the analysis even in the inelastic range of response, and is based on the initial elastic stiffness, the tangent stiffness damping force is proportional to instantaneous value of the stiffness and it is updated whenever the stiffness changes and it will be zero while the structure deforms along a yield plateau (Priestley et al., 2007-I). Thus the tangent stiffness damping can consider reduction in damping force as the structural stiffness softens following yield, and reduction in the energy absorbed by the elastic damping. This situation for Multi-Degree-of-Freedom (MDOF) analyses is often further confused by the adoption of the Rayleigh damping, which is a combination of mass-proportional and stiffness-proportional damping (Priestley et al., 2007-I; Priestley et al., 2007-II; Priestley and Grant, 2005).

Within the context of *Performance Based Seismic Design*, accurate prediction of non-linear response is essential to structural and non-structural control performances. While various variables such as strength, hysteretic response and earthquake characteristics can affect nonlinear response, the effect of the choice of viscous damping model on non-linear response using different hysteretic models in frame analysis is

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less obvious. However, many analysts still consider default models available in commonly available analysis codes, which is typically proportional to the initial stiffness, to be rather insignificant for either SDOF or MDOF inelastic analyses, as the effects are expected to be masked by the much greater energy dissipation associated with hysteretic response. Moreover, research by others has shown that the choice of damping model between the constant and tangent stiffness proportional could be significant particularly for short-period structures and as a consequence call into question well established displacement equivalence rules such as the equal energy and equal displacement approximation (Priestley et al., 2007-I; Priestley et al., 2007-II; Priestley and Grant, 2005).

In the research described in this study, a parametric study was conducted using over 100 real earthquake ground motions and five different hysteretic models to assess the impact of the choice of viscous damping model on the non-linear response of SDOF systems. In order to achieve multiple levels of non-linear response, several SDOF oscillators with different height, level of axial load and moment strength were considered with a wide range of ductility level. In total, the research presents the outcome of over 45,000 non-linear time history analyses.

NUMERICAL MODELS AND STUDY PARAMETERS

Various SDOF oscillators, representing reinforced concrete cantilever bridge piers were considered in the non-linear analyses. The SDOF cantilever columns all have the same diameter and properties but have different heights, longitudinal steel ratios and axial loads (See Fig. 1 and Tables 1-2) resulting in a wide range of initial periods between 0.40 to 3.30 sec. In total, 45 different SDOF oscillators were subjected to 100 different earthquake time histories using 5 different hysteretic models. All oscillators were analyzed using the initial stiffness proportional damping {ICTYPE 1 in *Ruaumoko* (Carr, 2004)} and the tangent stiffness proportional damping {ICTYPE 6 in *Ruaumoko* (Carr, 2004)}, as shown in Fig. 1.

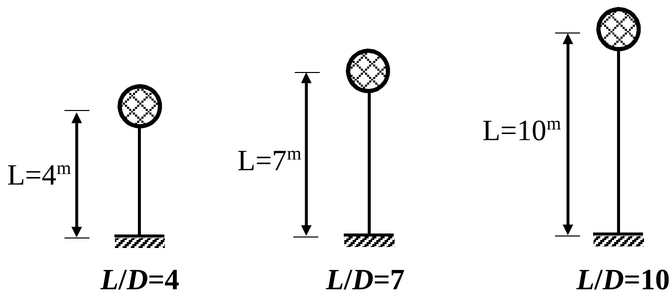

Figure 1. Aspect ratio of the considered SDOF oscillators

Table 1. The SDOF cantilever column models considered in non-linear analyses

	Aspect ratio (L/D)	**Number and diameter of long. rebar**	**Volumetric ratio of long. rebar $\rho_s = \sum A_s/D$**	**Volumetric ratio of trans. rebar $\rho_{sp} = 4A_{sp}/(D's)$**	**Axial load P (kN)**	**Damping model**
	4	8ϕ25	0.5%		$0.05f_c'A_g$=1178.1	Initial Stiffness Proportional
Column Models		16ϕ25	1.0%			
	7	24ϕ25	1.5%	0.9%	$0.10f_c'A_g$=2356.2	
		32ϕ25	2.0%			Tangent Stiffness Proportional
	10	40ϕ25	2.5%		$0.15f_c'A_g$=3534.3	

Table 2. Member and material properties

Parameter	**Value**
Cover	37^{mm}
Type of cross-section	Circular
Section diameter (D)	1000^{mm}
Diameter of transverse reinforcement	14^{mm}
Spacing of transverse reinforcement	75^{mm}
Type of transverse reinforcement	Spirals
Concrete compressive strength	30 MPa
Longitudinal and transverse reinforcement yielding strength	400 MPa
Longitudinal reinforcement maximum strength	600 MPa

Hysteretic Models

In the study, five hysteretic models, as shown in Fig. 2, were considered to investigate the effect of using the initial or tangent stiffness damping in the NTHA. The *Thin* (*Small*) *Takeda* and *Large Takeda* models represent reinforced concrete column and beam members, respectively. The *Ramberg-Osgood* model is appropriate for steel structures while the *Ring-Spring* (*Flag-shaped*) resembles a post-tensioned column or wall. The *Bi-linear* shape was included because of its importance in seismic analysis, and also can represent various types of isolation systems (Priestley et al., 2007-III).

Moment-curvature relationships for the RC column members were determined using Matlab code *CUMBIA* (Montejo and Kowalsky, 2007). The *CUMBIA* was developed for the design and analysis of RC members using unconfined and confined concrete models proposed by Mander *et al.* (1988) and the steel model proposed by King (1986). The parameters which define the smallest and largest loop area for the *Thin-Takeda* and *Large-Takeda* models were respectively selected as $\alpha=0.5$ $\beta=0$ and $\alpha=0$ $\beta=0.6$. The loop area parameter for the *Ring-Spring* was chosen as $\beta=0.35$. Note that this hysteretic shape is self-centering and has zero residual displacement, as shown in Fig. 2, which was obtained from the manual for the analysis code *Ruaumoko* (Carr, 2004).

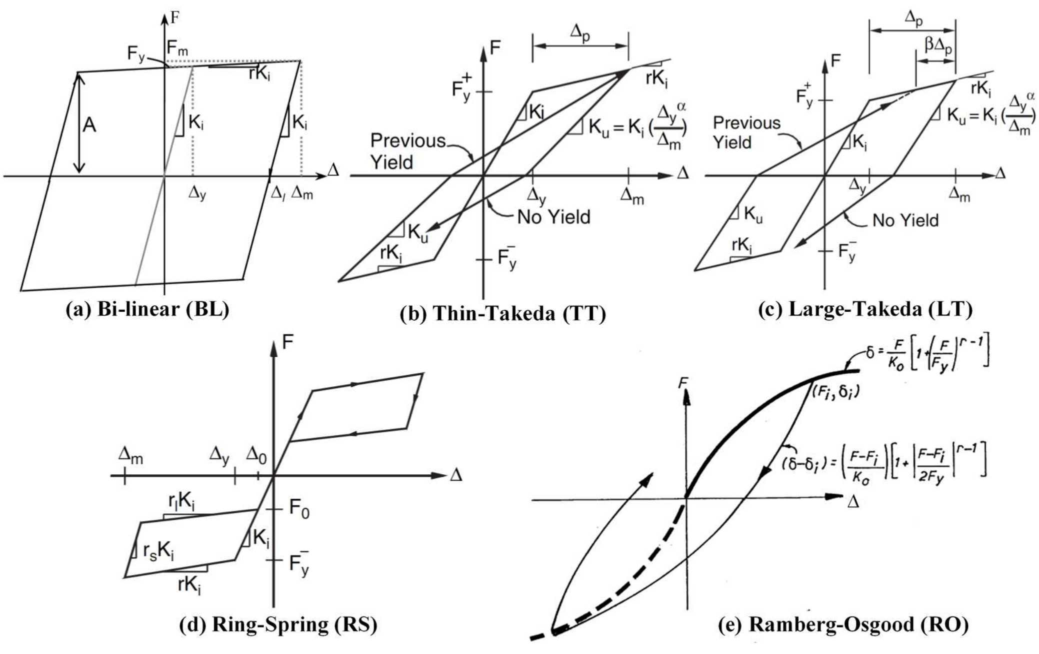

Figure 2. Hysteretic models considered in the study (Carr, 2004)

Earthquake ground motions

There is a tendency to use "real" earthquake records for parameter studies of this kind, as being more representative of expected response than will be the case with artificial records (Priestley and Grant, 2005). Thus, in order to investigate the effect of damping assumption (initial / tangent stiffness proportional) on displacement demand of the SDOF systems, a total of 100 ATC55 / FEMA440 real ground motions which were recorded for different types of soil with moment magnitudes ranging between 6.0 and 7.8 were used in the analyses. The ground motion records were categorized in five groups according to the local site conditions (site class B, C, D, E and near fault) at the recording station by Miranda (2002).

NON-LINEAR TIME HISTORY ANALYSES OF SDOF SYSTEMS

In the non-linear time history analyses (NTHA), the elastic damping which represents the damping in initial stages of cyclic response before hysteretic damping is activated, is assumed as 5% of critical damping. The *P-delta* effects and moment-axial load interaction were not included in the non-linear analyses. The NTHA were carried out using the program *RUAUMOKO* (Carr, 2004), using *Newmark Constant-Average Acceleration* integration with $\beta = 0.25$. In order to show the difference between the responses of two damping approximations on the SDOF systems, the peak displacement ratios predicted by the tangent stiffness damping model were compared to those obtained using the initial stiffness model for the each ground motion. The peak displacement ratios between the initial and tangent stiffness proportional damping models were computed by Eq.1.

$$r_\delta = \frac{\delta_{TS}^{Peak}}{\delta_{IS}^{Peak}} \quad (1)$$

where the displacement ratio δ obtained from the **T**angent **S**tiffness and **I**nitial **S**tiffness proportional damping is indicated by subscript "***TS***" and "***IS***", respectively (Fig. 3). As shown in Fig. 3, when the displacement demand using the tangent stiffness damping is larger than for the initial stiffness damping, the corresponding displacement ratio is greater than 1.00. Results of the NTHA are presented in the next section to demonstrate the impact of choice of damping model on the displacement ductility. The displacement ratios (δ_{TS} / δ_{IS}) calculated by the peak displacement demands of SDOF oscillators are discussed in terms of the *displacement ductility* (μ_Δ) and *residual drift* ($\Delta^{Res.}$). The effect of axial load levels and soil types of the earthquake ground motions were evaluated as well.

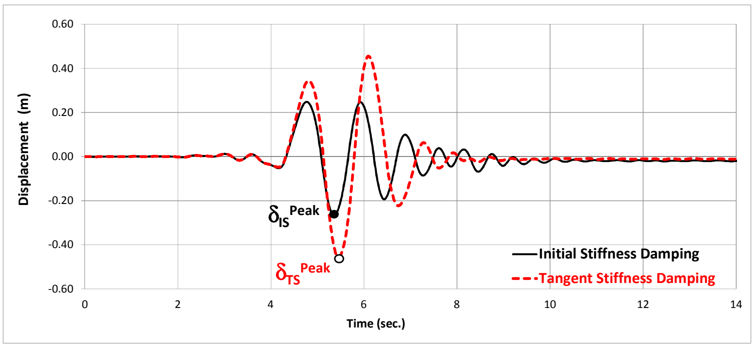

Figure 3. A sample displacement response using initial and tangent stiffness damping

ANALYSIS RESULTS AND IMPACT OF DAMPING MODEL ON DISPLACEMENT DUCTILITY

After conducting the NTHA of various SDOF oscillators subjected to the ground motion records, the displacement ratios (δ_{TS} / δ_{IS}) corresponding to the peak displacement demands were plotted with respect to the parameters below and specific trends in individual analyses are explained.

Displacement Response with Ductility Level

In determination of the displacement ductility μ_Δ of the SDOF cantilever systems, the response of initial stiffness damping models were used. The yield displacement was calculated by means of the Eqs. 2 and 3 (Priestley et al., 2007-III; Priestley, 1993).

$$\delta_y = \phi_y \, (H+L_{sp})^2/3 \tag{2}$$

$$\phi_y = 2.25\varepsilon_y \,/\, D \tag{3}$$

where $\boldsymbol{H}$ is the column height, $\boldsymbol{L_{sp}}$ is the effective additional height representing strain penetration effects, ε_y is the yield strain of the flexural reinforcement and $\boldsymbol{D}$ is the section depth for the circular column.

Fig. 4 presents the variations of displacement ratio with the ductility of the SDOF systems. As shown from Fig. 4, there is no difference on the displacement demands of the systems which respond fully elastic or slightly non-linear for ductility less than 2. It is apparent that the tangent stiffness damping models yields predominantly larger displacement when compared to the initial stiffness damping model as the ductility increases for all hysteresis rules considered. It should be noted that the difference between both damping approaches are more critical particularly in the period range of *0.40* to *0.85* sec. Depending on cyclic characteristic of the hysteretic shapes, the extreme displacement ratio obtained from the tangent stiffness damping model may be larger in a range of *1.7* to *4.0* times than for the initial stiffness, indicating significant influence and significantly non-conservative, as shown in Fig. 4. The *Ring-Spring* hysteresis model, which is of particular significance to pre-stressed structural systems, shows the largest difference between tangent stiffness and initial stiffness displacements. Although the displacement ratios r_δ tend to decrease as the initial period increases, the maximum ratios show constant tendency as the displacement ductility increases so that the trend is more apparent for all ductility levels larger than μ_Δ = 7 (Fig. 4).

For some column models which show slightly non-linear response (particularly $2 \leq \mu_\Delta \leq 5$), the analysis results indicate that for the hysteretic models which tend to produce levels of residual displacement, the initial stiffness damping yields maximum deformations which are often substantially less than those predicted by tangent stiffness damping. This means that the displacement demands using the initial stiffness damping model were larger than using the tangent stiffness model. One of the reason of this response is due to the cyclic characteristics of the hysteretic shape and residual displacement remaining in the system. Furthermore, individual analyses show that although characteristics of the displacement vs. time responses with respect to the corresponding cases are very compatible for both damping modeling, the initial or tangent stiffness damping model can be conservative depending on amount and sign of residual displacement remaining in the system. It should be also noted that this response is regardless from the ground motion type.

Effects of Axial Load and Site Classes of Ground Motions on Displacement Response

When determining of displacement responses of the SDOF systems, the level of axial load and site class of the earthquake ground motions were are also discussed in the study. The graphics corresponding to displacement ratios for both parameters are presented in Figs. 5 and 6 for only the *Bi-Linear* hysteretic model for the sake of brevity. It should be noted that all evaluations performed with regard to these parameters are also consistent for other hysteretic models considered in the study. Referring to Fig. 5, the analysis results indicate that changes in the level of axial load has no significant effect on the displacement response. The displacement ratios of the SDOF systems, which have nearly the same period or ductility range, show similar response for different levels of axial load. Furthermore, as the site class of the ground motion records changes, the displacement ratios are significantly affected, as shown from Fig. 6. The displacement ratios determined from the *near-fault records* are dramatically larger than others since the largest ductility demands occur during these analyses, as would be expected.

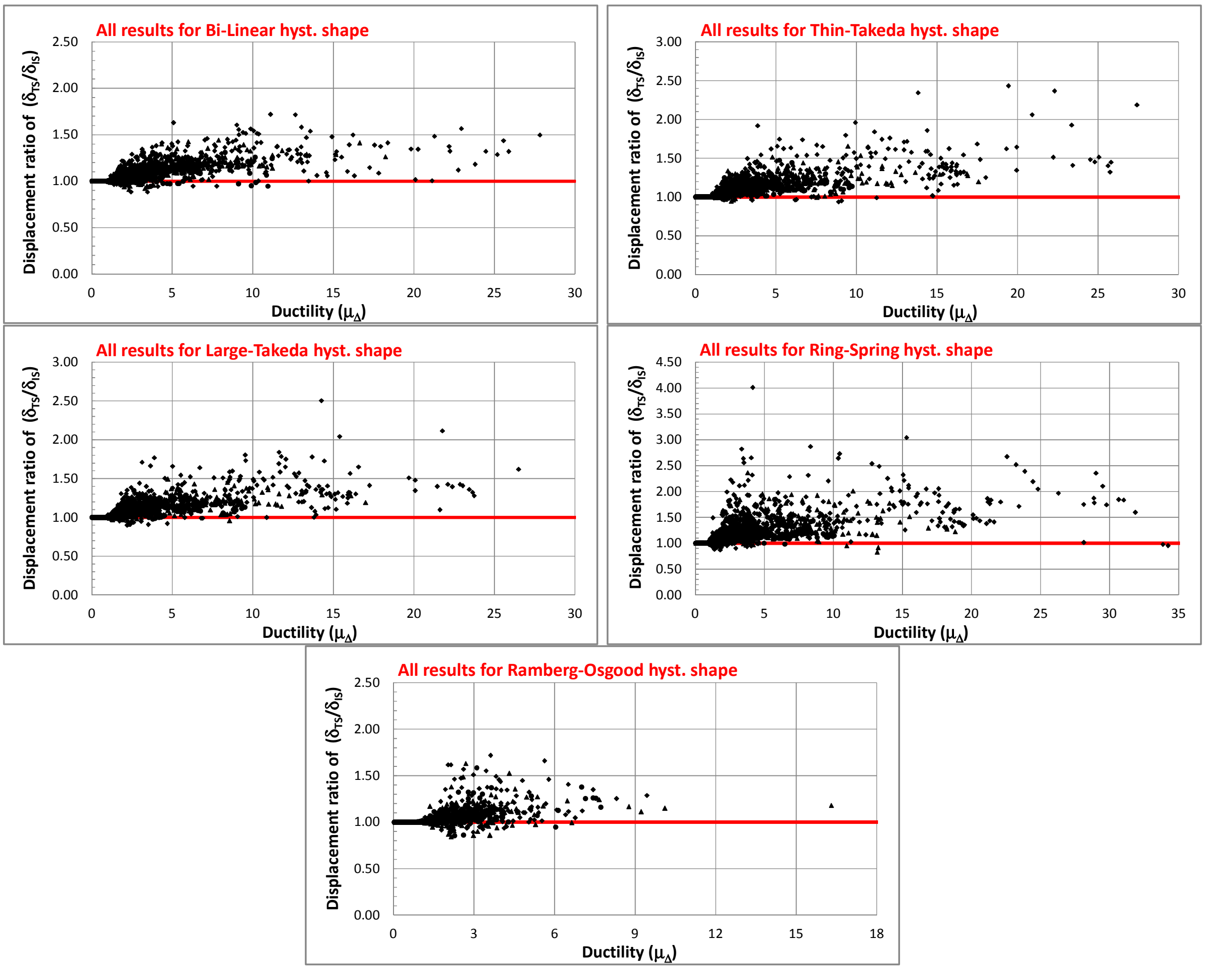

Figure 4. Variations of the displacement ratios (δ_{TS} / δ_{IS}) with ductility for SDOF oscillators

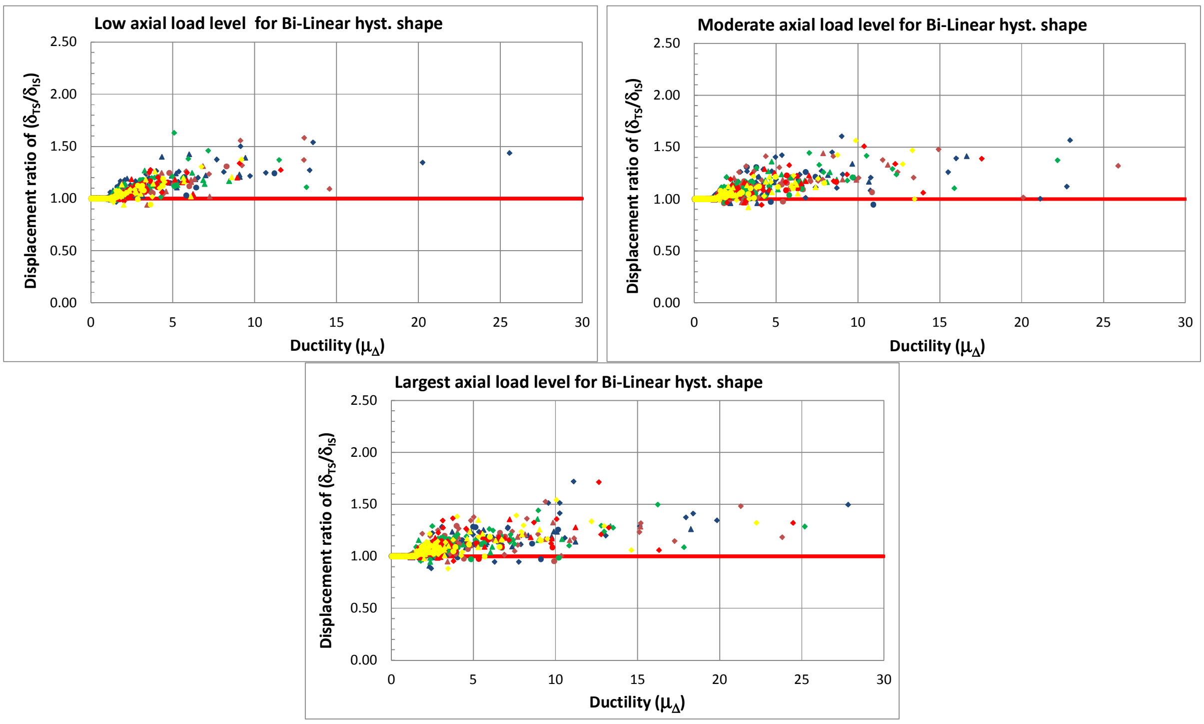

Figure 5. Effect of different axial load levels to displacement ratios (Bi-linear model)

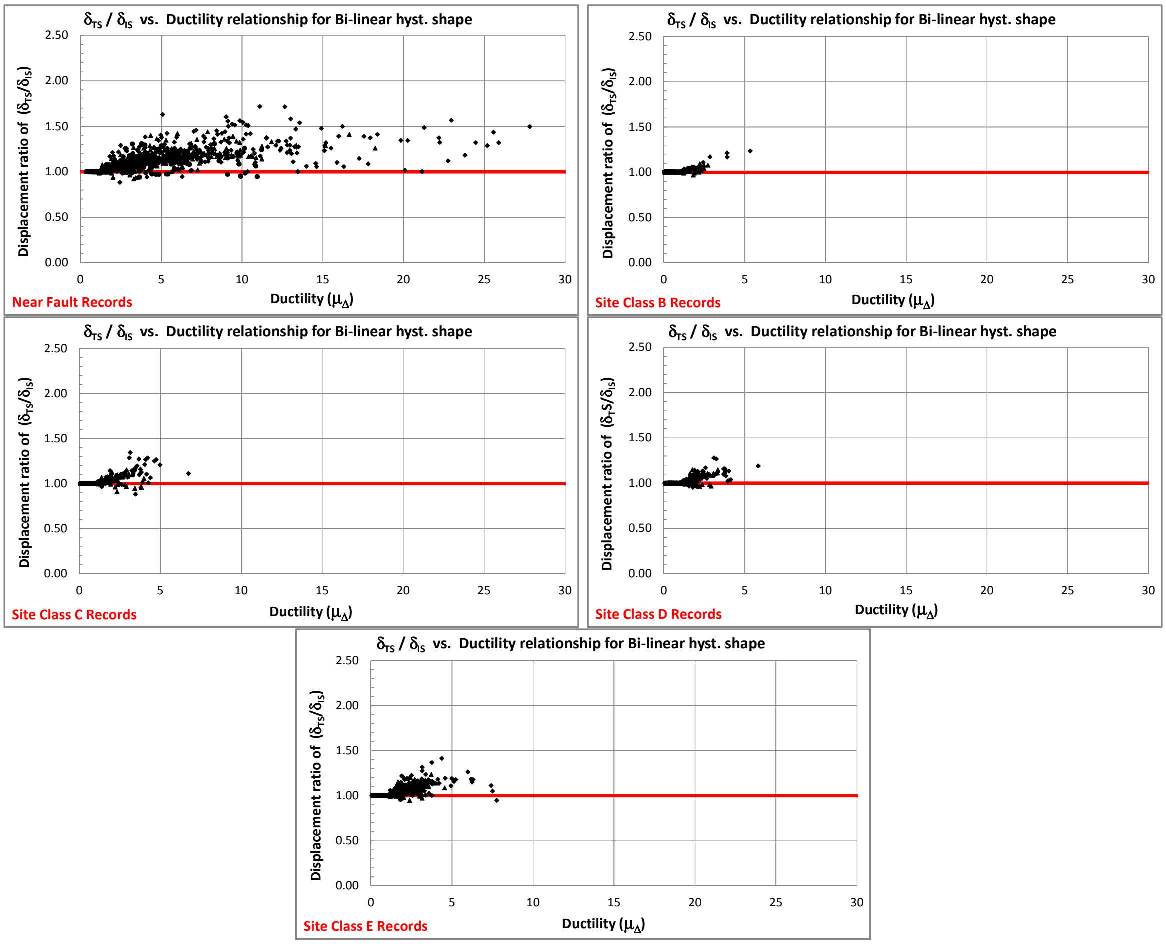

Figure 6. Effect of soil types of ground motion records to displacement ratios (Bi-linear model)

Effect of Residual Displacement on Displacement Response

The displacement ratios determined for some column models, as noted previously, which respond fully elastic or slightly non-linear during the ground motion records, can be lower than 1.00. The individual analysis results showed that the residual displacement remaining in the system can affect the displacement ratio. To further investigate this behavior, the variation of displacement ratio with residual drift, based on both the initial and tangent stiffness damping models ($\Delta_{IS}^{Res.} = \delta_{IS}^{Res.} / L$ and $\Delta_{TS}^{Res.} = \delta_{TS}^{Res.} / L$), are given in Fig.7 for all hysteretic models considered. It is interesting to note that lower residual drifts tend to give a wider gap between tangent and initial stiffness damping results. This is not surprising as large residual drifts can skew the peak response values dramatically as a system oscillates about an offset position.

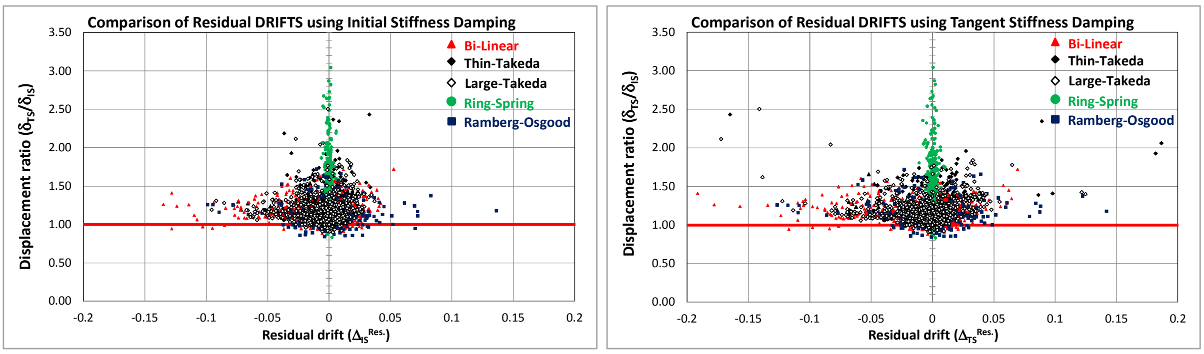

Figure 7. Comparison of displacement ratio δ_{TS}/δ_{IS} with residual drift relationships

CONCLUSIONS

The choice of damping model is actually more impactful than the amount of damping assumed in analysis. The impact of damping model have a large impact on the displacement response of non-linear systems. This is of particular importance for assessing performance objectives within the context of *Performance-Based Seismic Design* where an accurate estimate of displacements of inelastic systems is important. Given that research by others has suggested that tangent stiffness damping more accurately predicts the actual structural response for yielding systems, use of initial stiffness damping for analysis becomes even more troubling. The damping force (the initial or tangent stiffness proportional) is dependent on cyclic characteristic of the hysteretic shapes. For the hysteretic models with large potential residual displacements the influence of hysteretic characteristics have important impact on the structural demands, but trends are less obvious but still present.

Presented in this paper are the analysis results of several SDOF oscillators subjected to a series of 100 ground motion records. Analyses were conducted using both initial and tangent stiffness proportional damping. The outcome of the analyses are consistent with prior studies which indicate that the tangent stiffness proportional damping should be used to predict non-linear response of systems. This is an important outcome as analysts often use initial stiffness damping model due to limitations of the software used to conduct such analysis, and a lack of data showing the potential impact that the choice of damping model can have. Although the analyses were conducted for only SDOF systems, it is thought that similar results would be obtained for MDOF systems. Further studies are underway to investigate this, as well comparisons to additional experimental shake table data, where available.

ACKNOWLEDGMENTS

The work of the first author was financially supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the program BIDEB–2219. The research described in this paper was conducted in the Department of Civil, Construction and Environmental Engineering at North Carolina State University, Raleigh, NC, USA.

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