Due September 30^{th} 2011

FFZ5135 Statistical Mechanics
Homework # 1

1. In throwing two dice, you may throw a sum n which ranges from $n = 2$ to $n = 12$.
   (a) Find the probability distribution function $P(n)$ for throwing a sum n with two dice.
   (b) Calculate the mean value $\overline{n}$ of the sum thrown.

2. In the one-dimensional random walk the probability that after N steps there will be R steps to the right and L steps to the left is given by

$$P_N(R, L) = \frac{N!}{R!\,L!}\,p^R\,q^L$$

   Here p and q are the a-priori probabilities for a right and a left step. The mean value for R is $\overline{R} = Np$, while the mean value for L is $\overline{L} = Nq$.
   (a) Show that $\overline{L^2}$, the mean square value of the number of left-hand steps, is $\overline{L^2} = \overline{L}^2 + Npq = N^2q^2 + Npq$.
   (b) If we define the variable h as

$$R = \overline{R} + \frac{h}{2} \qquad\qquad L = \overline{L} - \frac{h}{2}$$

   then the displacement after N steps is $(R - L)$=$(\overline{R} - \overline{L}) + h$. Show that $\overline{h^2} = 4Npq$.

3. Gas pressure and molecular collisions against the containing vessel.
   (a) Show that the mean momentum Δp_x transferred to the containing walls of a box per colliding gas molecule is $\Delta p_x = \sqrt{4mkT}$ (Use equipartition theorem.)
   (b) Express the pressure on the wall in terms of the mean momentum transfer Δp_x and the number of collisons/cm^2 sec (dN/dt).
   (c) Assume that oxygen is in a container at 300°K and exerts a pressure of 1 atmosphere. Compute numericaly (dN/dt) he number of collisions/cm^2 sec between the oxygen molecules and the wall.
   (Use k=1.38×10^{-16} $ergs$/°K, 1 atmosphere=10^6 $dynes/cm^2$, and m_{O_2}=5.31×10^{-23} gr.)

4. From the macroscopic definition of entropy, show that
   (a) It is a state function for an ideal gas.
   <u>Hint.</u> Combine the first and second laws of thermodynamics and remember that $dE = nc_v dT$ for an ideal gas.

   (b) If a substance at a temperature T_1, is brought into thermal contact with an equal amount of the same substance at a temperature $T_2 < T_1$, the final equilibrium temperature must satisfy $T_1 > T > T_2$. What would the final temperature T be if
      i. heat is spontaneously transferred,
      ii. a reversible heat engine is operated,

   between the two substances as reservoirs until thermal equilibrium is reached.