Using computer algebra systems in mathematical classrooms

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Abstract This paper describes a research whose main focus is the use of Computer Algebra Systems (CAS) in mathematical classrooms and the didactical possibilities linked with its use. The possibilities of integrating Self-Regulated Learning (SRL) within the CAS environment are brought into focus. Forty-three Israeli students (mean age 13.3) were assigned to two learning algebraic groups. The first group received explicit meta-cognitive SRL with CAS (CAS + SRL); the second group was exposed to CAS without SRL (CAS). Empirical results from the experimental and case study designs revealed that (CAS + SRL) students outperformed (CAS) students on algebraic thinking and that (CAS + SRL) students regulated their learning more effectively.

Keywords: Algebra; Computer Algebra System; Control group; Discourse; Empirical; School; Self-regulated learning; Performance

Introduction

Algebra has emerged as one of the central themes of the Principles and Standards for School Mathematics [National Councils of Teachers of Mathematics (NCTM, 2000)]. In addition, algebra continues to be an essential component of contemporary mathematics and its applications in many fields. Algebra is frequently described as ‘generalised arithmetic’, and indeed, algebraic thinking is a natural exertion of arithmetical thinking. Both arithmetic and algebra are useful for describing important relation- ships in the world. But although arithmetic is effective in describing static pictures of the world, algebra is dynamic and a necessary vehicle for describing a changing world.

To think algebraically, one must be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts… working with patterns is at the heart of mathematics. Exploring patterns is a vehicle to provoke thinking about variables and functions. (Friel et al., 2001, p.2)

Research (e.g. Sfard & Linchevski, 1994; Usiskin, 1996) shows that learning algebra appears to be more complex than previous thought. Students face considerable difficulties in moving from arithmetic to algebra. Generally speaking, research reveals that a large percent of students are unable to cope with algebraic letters as
unknown or generalised numbers (e.g. Kuchemann, 1981; Macgregor & Stacey, 1997). The main purpose of the present study is therefore to investigate instructional methods that have the potential to enhance students’ algebraic thinking.

During the last decade the availability of CAS environments has increased dramatically. The interaction with the computerised system has promoted the practice of using algebraic language for symbolic and numerical computation. This interaction has encouraged the understanding of algebraic reasoning, looking for patterns, and reflection on the solution process. Research on the use of CAS has indicated evidence of success as well as difficulties (e.g. Monagham, 1994; Lagrange, 1996). Lagrange (1996) state that CAS has a great potential to improve student learning in mathematics, but easier calculation did not automatically enhance students’ reflection and understanding. He recommends more ‘direct’ teaching on how to think. Lagrange concludes that much fieldwork is required to establish the conditions under which the potential of computer algebra can be actualised.

Recently, there has been much interest in the role of metacognition in mathematics education (e.g. Hacker, 1998; Mevarech & Kramarski, 1997; OECD, 2000; Kramarski & Ritkof, 2002). Research in metacognition has focused on students’ self-regulated skills in addition to classical subject matter knowledge. Self-regulated learning (SRL) is a vital prerequisite for the successful acquisition of knowledge in school and beyond, and is thus of particular importance in lifelong learning.

What young people will need to know as adults, it seems appropriate to assume a dynamic model of continuous acquisition of new knowledge and skills. Self-regulated learning is to be seen as a central element in this dynamic model of knowledge acquisition. (Baumert et al., 2000, p.2)

The concept of self-regulation skills means having the ability to develop metacognitive knowledge, attitudes, and control behaviours, which enhance and facilitate future learning. Abstracted from the original learning context, these skills can be transferred to other learning situations. Researchers (e.g. Schoenfeld, 1985; Mevarech & Kramarski, 1997; Kramarski et al., 2001; Kramarski & Mevarech, in press) note that features of self-regulated behaviours can be learned through practice and reinforcement. In particular, the method of Mevarech & Kramarski (1997), called IMPROVE, emphasises the importance of providing each student with the opportunity to construct mathematical meaning by involving themselves in mathematical self-regulated learning. The IMPROVE method is based on self-questioning via the use of metacognitive questions that focus on: (a) comprehending the problem (e.g. What is the problem all about?); (b) constructing connections between previous and new knowledge (e.g. What are the similarities/differences between the problem at hand and the problems solved in the past? and why?); (c) using appropriate problem solving strategies (e.g. What are the strategies/tactics/principles appropriate for solving the problem and why?); and in some studies (d) reflecting on the processes and the solution (e.g. What did I do wrong here?; Does the solution make sense?).

Evidence shows positive effects of self-regulated learning employed in non-computerised environments (e.g. Mevarech & Kramarski, 1997; Kramarski et al., 2001; Kramarski & Mevarech, in press) as well as the effects of using such pedagogy in computerised environments (e.g. Teong et al., 2001; Kramarski & Zeichner, 2001; Kramarski & Ritkof, 2002). Effects have been found on mathematical reasoning, giving mathematical explanations and promoting mathematical discourse. Given these studies, there is reason to believe that providing SRL within the CAS
environment will enrich the use of metacognitive strategies, and thereby improve students’ algebraic thinking.

Moreover, there is reason to suppose that different instructional methods may affect students’ self-regulated skills differently. Students who are exposed to metacognitive SRL are expected to be better able to reflect on solution processes than students who are not exposed to such learning. In addition, discussing metacognitive issues with others is expected to enhance self-regulated skills.

The purpose of the present study is to investigate the differential effects of (CAS) and self-regulated learning on algebraic thinking and self-regulated skills. In particular, the study compares two instructional methods: CAS learning embedded within self-regulated learning (CAS + SRL), and CAS learning without CAS.

**Method**

**Participants**
Participants were 43 eighth-grade students (boys and girls) who studied in two classrooms, randomly selected from two junior high schools. The schools were similar in terms of size and average level of socio-economic status, as defined by the Israel Ministry of Education. The students’ mean age (13.3 year-old) was similar.

**Treatments**
All classrooms studied algebra five times a week during a five-month period according to the mathematics curriculum developed by the Israel Ministry of Education. In all classrooms, students studied how to: (a) use algebraic symbols; (b) manipulate algorithms (c) transform verbal expressions into algebraic expressions; (d) develop solutions of standard tasks/problems; and (e) develop problem solving strategies through the use of tables, graphs, and verbal expressions.

All students practised the same tasks, and used the same textbook. Classrooms were randomly assigned to one of the following two groups:

**CAS + SRL group:** In this group, students were exposed under this condition to self-regulated learning embedded within DERIVE algebra software. With this software, the student can practice the algebraic language for symbolic and numerical computation. In particular, the software enables the student to substitute variables, simplify algebraic expressions and solve equations. The Appendix contains an example of such activities. Each week students studied for four hours in the classroom and one hour in the computer laboratory (20 lab hours in total). The SRL was based on the IMPROVE’s self-questioning method that emphasises the use of comprehension, connection, and strategy and reflection questions (Mevarech & Kramarski, 1997). The self-regulated questions were used by each individual student when their turn arrived to solve a problem/task aloud, and by the teacher when she introduced the new concepts to the whole class, reviewed the lesson at the end of the class, and provided help to the students.

**CAS group:** Students in this group studied with CAS but were not exposed to self-regulated learning. Each week the CAS group learned four hours in the classroom and one hour in the computer laboratory (20 lab hours in total).

**Measurements**
Four types of measurements were used to assess students’ mathematical reasoning.
and self-regulated learning: Mathematics prior knowledge; algebraic thinking; metacognitive knowledge; and cognitive-metacognitive behaviours. For simplicity all scores on the mathematics tests are presented as percentages of correct answers.

Mathematics prior knowledge

A 21-item pre-test was administered to all students at the beginning of the school year. The test covered operations with positive and negative numbers, order of operations, the basic laws of mathematics operations, algebraic expressions, and open-ended computation problems.

**Scoring:** For each item, students received a score of either 1 (correct answer) or 0 (incorrect answer), and a total score ranging from 0 to 21. The Kuder-Richardson reliability coefficient was $\alpha = 0.87$.

Algebraic thinking

The post-test included 17 items that are based on the following four criteria:

(a) Manipulating Algorithms – five items (e.g. solution of equations, operations with algebraic expressions);

(b) Symbolic Reasoning – four items (e.g. what can you say about $r$ if: $r = s + t$ and $r + s + t = 30$).

(c) Exploring Patterns four items (e.g. the following design is composed from white and black squares, if you know the number of the black squares can you find the number of the white squares? If the number of the black squares is K find the pattern for the number of the white squares); and

(d) Analysing Changes – four items (e.g. which changes faster: $2n$ or $n + 2$, explain).

**Scoring:** For each item, students received a score of either 1 (correct answer) or 0 (incorrect answer), and a total score ranging from 0 to 17. The Kuder-Richardson reliability coefficient was $\alpha = 0.87$.

Metacognitive questionnaire

The questionnaire included 17 items that were adapted from the studies of Montague & Bos (1990) and Kramarski & Mevarech in press). The questionnaire assessed students’ metacognitive knowledge regarding their specific use of strategies while working in a computerised environment (e.g. It is easier for me to investigate mathematical rules in the CAS environment).

**Scoring:** Each item was constructed on a 5-point Likert type scale ranging from 1 (never) to 5 (always), and a total score ranging from 17 to 85. Cronbach’s alpha reliability coefficient was $\alpha = 0.86$.

Cognitive-metacognitive behaviours

Qualitative analysis was conducted on students’ problem solving ‘think-aloud’ protocols. A pair of students was selected randomly from each group for a case study design. Students were asked to solve collaboratively a mathematical transfer task, which was not apart off their classrooms instruction.

The think-aloud protocols were analysed with Schoenfeld’s (1985, pp. 297–301) episode analysis for cognitive-metacognitive behaviours. The protocols were parsed into episodes, representing the following six problem solving behaviours: reading, analysing, exploring, planning, implementing and verifying. Reading refers to reading the task/problem or other instructions; analysing refers to understanding the
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task, the givens and what has to be found; exploring refers to using different
representations, such as drawing; planning refers to using different strategies for
solving the task/problem, such as using variables, guessing the solution;
implementing refers to calculations and using algorithms; and verifying refers to
evaluating the solution. According to Schoenfeld (1985), cognition is involved in
doing, and metacognition is involved in choosing and planning what to do and
monitoring what is performed. Cognitive behaviours are reading, exploring, and
implementing, whereas analysing, planning and verifying are metacognitive, which
are indicators of the ability to control and monitor the problem solving process.

Procedure
The study focused on teaching algebra in all classrooms for a five-month period. At
the beginning of the school year, all students were given the pre-test and the
metacognitive questionnaire. Then, at the end of academic year the post-test and the
metacognitive questionnaire were administered. In addition, ‘think aloud’ protocols
of two pairs were analysed, one pair from each group.

Results
Algebraic thinking
The first purpose of the study was to investigate the differential effects of two
learning groups: CAS + SRL and CAS on students’ algebraic thinking.

Table 1 contains the mean scores and standard deviations on measures of
algebraic thinking by time and treatment.

Table 1. Scores on an algebraic thinking test.

<table>
<thead>
<tr>
<th></th>
<th>CAS+SRL n = 20</th>
<th>CAS n = 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test m</td>
<td>72.40</td>
<td>73.61</td>
</tr>
<tr>
<td>s.d.</td>
<td>14.59</td>
<td>11.64</td>
</tr>
<tr>
<td>Post-test m</td>
<td>76.10</td>
<td>62.61</td>
</tr>
<tr>
<td>s.d.</td>
<td>17.01</td>
<td>18.80</td>
</tr>
</tbody>
</table>

Note. Scores are in percentages

Table 2 contains the mean scores, standard deviations, median, minimum,
maximum and range scores by criteria of algebraic thinking and treatment.

Table 2. Scores by criteria of an algebraic thinking test and treatment.

<table>
<thead>
<tr>
<th></th>
<th>CAS+SRL n=20</th>
<th>CAS n=23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Manipulating algorithms</td>
<td>11.90</td>
<td>10.25</td>
</tr>
<tr>
<td>s.d.</td>
<td>2.38</td>
<td>1.74</td>
</tr>
<tr>
<td>Median</td>
<td>12.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Min</td>
<td>6.00</td>
<td>6.00</td>
</tr>
<tr>
<td>Max</td>
<td>15.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Range</td>
<td>9.00</td>
<td>6.00</td>
</tr>
<tr>
<td>Symbolic reasoning Exploring patterns Analysing changes</td>
<td>5.30</td>
<td>0.99</td>
</tr>
<tr>
<td>s.d.</td>
<td>0.98</td>
<td>1.85</td>
</tr>
<tr>
<td>Median</td>
<td>11.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Min</td>
<td>8.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Max</td>
<td>15.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Range</td>
<td>7.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Exploring patterns Exploring patterns Analysing changes</td>
<td>3.00</td>
<td>7.00</td>
</tr>
<tr>
<td>s.d.</td>
<td>3.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Median</td>
<td>12.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Min</td>
<td>6.00</td>
<td>6.00</td>
</tr>
<tr>
<td>Max</td>
<td>15.00</td>
<td>15.00</td>
</tr>
<tr>
<td>Range</td>
<td>7.00</td>
<td>7.00</td>
</tr>
</tbody>
</table>

Note. Scores are in percentages

Results indicate that prior to the beginning of the study no significant differences
were found between the two learning groups ($F_{1,41} = 1.20, p > 0.05$) on mathematical
prior knowledge. One-way MANOVA on the four criteria of the post-test scores as
dependent variables (Manipulating Algorithms, Symbolic Reasoning, Exploring
Patterns, and Analysing changes) indicated significant differences at the end of the
study between the two learning groups ($F_{4,38} = 3.89, p < 0.01$).

Further analysis (ANOVA) on each dependent variable indicated significant
differences between the learning groups on two criteria of algebraic thinking: Symbolic Reasoning ($F_{1,41} = 12.76, p < 0.001$) and Exploring Patterns ($F_{1,41} = 6.59, p < 0.01$). It was found that the CAS + SRL students significantly outperformed the CAS students on these measures. No significant differences between the learning groups were found on the other two criteria of algebraic thinking: Manipulating Algorithms ($F_{1,41} = 0.23, p > 0.05$) and Analysing Changes ($F_{1,41} = 1.76, p > 0.05$).

**Self-regulated skills**
The secondary purpose of the study was to compare the differences in SRL skills among the two learning groups. These skills were investigated by analysing metacognitive knowledge and cognitive-metacognitive behaviours.

**Metacognitive knowledge**
Table 3 presents the mean scores, adjusted means and standard deviations on metacognitive knowledge by treatment and time. Results indicated that prior to the beginning of the study no significant differences were found between the two groups on metacognitive knowledge ($F_{1,41} = 2.90, p > 0.05$). ANCOVA with the pre scores as a covariant indicated significant differences between the two groups on the post-test ($F_{1,40} = 15.28; p < 0.0001$). The study found that the CAS + SRL students significantly outperformed the CAS students on metacognitive knowledge regarding the use of computerised environment.

**Cognitive-metacognitive behaviours**
The following task shows a qualitative analysis procedure on ‘think aloud’ protocols in solving a transfer task. The task was adapted from NCTM (2000, p. 268).

**The transfer task:**

A certain rectangle has length and width that are whole numbers of inches, and the ratio of its length to its width is 4 to 3. Its area is 300 square inches. What are its length and width?

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**CAS+SRL group: Student A and Student B**

1. **A & B:** Read the problem aloud (reading).
2. **A:** May we use variables? (planning).
3. **B:** What are the givens? What should we find? (analysing).
4. **A:** The length and width of the rectangle (analysing).
5. **B:** Draw a rectangle and assign the variables X and Y on the length and width of the rectangle (exploring).
6. **A:** Maybe we need two equations? (planning).
7. **B:** OK, let’s solve the problem with two different equations: $300/x = y; 3x*4y = 300$ (planning).
8. **A & B:** Verify the result. It is a mistake (verifying).
9. **A & B:** Read again the problem, and try to explain what does ratio mean (reading & analysing).
10. **B:** May we guess the answer? (planning).
11. **A & B:** B reviews his drawing, and A also draws a rectangle (exploring).
12. **A & B:** They try to find numbers that have the appropriate ratio 3:4.
In the first trial they multiply each component with the number 3:
3*3 = 9; 4*3 = 12 (exploring & implementing).

13. A & B: Both students verify the solution 12*9 = 108. This answer is incorrect (verifying).

14. A & B: They try once again, to multiply each component by the number 5:
3*5 = 15; 4*5 = 20; 20*15 = 300 (exploring & implementing).

15. A & B: This is a correct answer (verifying).

16. A & B: Let’s try to find the solution in another systematic way (planning).

17. A: We should try again to use variables (planning).

18. A: Writes an equation: 3x*4x = 300 (implementing).

19. A: Continues to solve the equation:
12x^2 = 300; x^2 = 25; x = 5
Conclusion: The length of the rectangle is: 3*5 = 15; the width is: 4*5 = 20 (implementing).

20. A: Verify the solution: 15*20 = 300, it is a correct answer (verifying).

The protocol shows that students who were exposed to CAS embedded within SRL in mathematical classrooms successfully obtained a correct solution. These students implemented a variety of cognitive-metacognitive behaviours during their problem solving process. The solution process indicates 20 moves, while most behaviours were metacognitive (three analysing, six planning and four verifying) and fewer were cognitive behaviours (two reading, four exploring and four implementing).

**CAS group: Student C and Student D**

1. C & D: Read silently the problem (reading).

2. It seems that the numbers 3 and 4 are the sides of the rectangle (analysing).

3. C: Which rectangle does it mean? (She draws a rectangle) (exploring).


5. Teacher: Explained what ratio means and practised some ratio examples.

6. C: First trial, x(x + 1) = 300 (implementing).

7. D: We have to multiply both (sides) with the same number (implementing).

8. D: Writes the equation 3x + 4x = 300, verifies the solution, and concludes that it is wrong (implementing and verifying).

9. C: The equation 3x + 4x = 300 is correct, because x is the number that we have to multiple for each one (side). The students verify the solution, and find that it could not be the correct solution because it is not a whole number (implementing and verifying).

10. Teacher: What does the number 300 mean?

11. C: Ha!!! it is the area, we have to multiply: 3x*4x = 12X (implementing).

12. Teacher: Remember that x*x = x^2

13. A: Try again 12 + x^2 = 300 (implementing)
   x^2 = 288

14. A: It is wrong (verification).

15. C: No, we have to do it that way (verifying).
   3x*4x = 300; 12x^2 = 300; x^2 = 25
They did not know how to continue.

The protocol shows that although CAS students implemented most types of their cognitive-metacognitive behaviours during the problem solving procedure, these students did not obtain a final correct solution. Their solution process indicates 15 moves, while most behaviours were cognitive (one reading, one exploring and six implementing); several behaviours were metacognitive (two analysing, three verifying and no planning). Figures 1 and 2 summarise the percent of cognitive and metacognitive behaviours that were implemented in each group. The percent was calculated, in each group, by dividing the amount of cognitive/metacognitive behaviours with the total behaviours in that group. The figures indicate that 57% of
the behaviours in the CAS + SRL group are metacognitive, whereas only 38% of the behaviors in the CAS group are metacognitive.

Summary of qualitative data
Both groups focused on the task and tried to solve it cooperatively. Yet, comparing the two discourses shows significant differences between the groups on the quality of the algebraic thinking and self-regulated learning.

(a) Using Mathematics Concepts: The CAS + SRL group used more often and richer formal mathematical concepts than the CAS group. The CAS + SRL group based their arguments on concepts such as: variables, givens, and equations. Unlike the CAS + SRL group, the CAS group based their arguments primarily on computations. This group also experienced difficulties in understanding the terms of ‘ratio’ and ‘area.’

(b) Using Symbolic Reasoning and Manipulating Algorithms: The CAS + SRL group manifested more symbolic reasoning based on the terms ‘ratio’ and ‘area.’ (‘Maybe we need 2 equations?’; ‘We should try again to use variables.’) They also manipulated correctly mathematical algorithms [lines 18–20]. The CAS group, although exposed to the same curriculum, manipulated mistakenly mathematical algorithms that were incorrect [line 11].

(c) Using Strategies: Generally speaking, the CAS + SRL students used a richer repertoire of strategies than CAS students. The CAS + SRL students developed the need to support their solution by using strategies, such as: drawing a rectangle and marking the variables on it [line 5]; substituting equations in different formats, first with two unknowns, and then with one unknown equation [lines 7 and 18]; guessing the numbers that have the appropriate ratio 3 : 4 and using a numerical-computation strategy to verify the solution [lines 12, 13, and line 18]. However, the CAS students primarily based their solutions on the drawing strategy [line 3] and in particular, on substituting equations with one unknown [lines 9, 11, 13, and 15].

(d) Self-Regulated Learning: The CAS + SRL students worked without teacher input during the solution process, whereas the CAS students depended on their teachers’ help. Generally speaking, the CAS + SRL students more often used metacognitive behaviours than CAS students. The CAS + SRL students guided their solution process with self-questioning (‘May we use variables’? [line 2]; ‘What are the givens’?; ‘What should we do?’ [line 3]; ‘May we guess the answer’? [line 10]). This is in contrast to the CAS students who throughout the entire discourse referred mostly to implementation statements [e.g. We have to multiply both (sides) with the same number’ [line 7].

Discussion
The present study investigates the differential effects of CAS embedded within Self-
Regulated Learning (CAS + SRL vs. CAS) on algebraic thinking and self-regulated skills. Although both environments focus on promoting algebraic thinking, the study found that students who were exposed to CAS embedded within Self-Regulated Learning (CAS + SRL) facilitated more positive effects on different aspects of algebraic thinking than students who were exposed only to CAS. The effects were observed on Symbolic Reasoning and Exploring Patterns. The study found no differences between the groups on Manipulating Algorithms and Analysing Changes. In addition, the study found that the CAS + SRL students outperformed their counterparts on metacognitive knowledge regarding the computerised environment, and that students better regulated their learning more effectively. Yet, the generalisations of the findings might be limited for at least three reasons. First, the sample involved only 43 students and two teachers. Second, only two ‘think aloud’ protocols were analysed, one for each learning group, and finally, only one task was used as a transfer task. Future research may need to investigate the effects of self-regulated learning on algebraic thinking by using a larger sample various types of tasks, and different types of measurements.

Two factors may explain the findings of this study. First, students who are trained to reflect on their problem solving processing probably focused on the information provided in the problem. The self-addressed comprehension questions (e.g. ‘What is the task/problem all about?’) probably guided students to look for all the relevant information. Also the connection questions (e.g. ‘How is this problem/task different/similar from what you have already solved?’) might lead students to pay attention to all the information and the structure of the task. Therefore, there is reason to believe that students who use the self-addressed questions referenced all the variables and data provided in the task.

The use of connection questions requires further consideration. Although SRL was employed during the regular study of algebra, the effects of SRL was assessed also on a transfer task to which students were exposed only in the interview periods, it is possible that the connection question does not only give cognitive tools to students for solving tasks employed during the studying, but also strengthens students’ self-confidence to approach new tasks. These conclusions support other research (Garofalo & Lester, 1985; Schoenfeld, 1987; Kramarski et al., 2001), which indicates that students’ problem-solving failures are often due to the ineffective use of what they know, and not due to a lack of mathematical knowledge. Successful problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems. Such metacognitive skills are much more likely to be developed in a classroom environment that supports these skills.

Of particular interest here are the findings that students who were exposed to SRL were more effectively able to transfer their algebraic thinking to a new task — a task that they were not exposed to in class. The study found that CAS + SRL students used more easily and correctly generalised letters (variables) and algebraic manipulations. Perhaps the poor performance of the CAS students may have stemmed from the limited metacognitive techniques employed in solving word problems. These students did not plan the steps in solving the problem, and implemented incorrect calculations. Schoenfeld (1987) also reported that these unsuccessful problem solvers, who exhibited limited metacognitive awareness, mathematical problem solving is often seen as a system of taking one step at a time,
without understanding the general principle of the problem.

There is reason to believe that SRL facilitated metacognitive skills. This in turn affects algebraic thinking and students’ ability to transfer their knowledge to new situations. Masui & De Corte (1999) and Kramarski et al. (2001) found that students who were exposed to metacognitive training had more knowledge about orienting and self-judging themselves than students in the control groups. In these studies, both meta-knowledge and transfer behaviour were positively related to academic performance. Further research based on systematic observations may identify other measures of metacognitive skills, so more empirical evidence can be collected regarding the relation between SRL and metacognitive skills.

Finally, the testing situation of the transfer task in the present case study involved only one context. Additional research is needed to investigate students’ ability to solve problems representing different contexts. Embedding such research with both clinical interviews and video filming may provide deeper insight into our understanding of how students solve problems. This is particularly important in light of current research showing the effects of contexts on students’ ability to solve mathematical problems (e.g. Mevarech & Stern, 1997; OECD, 2000).

Conclusions
The study raises the question: How can computer technology be used to enhance cognitive development in mathematics? The findings indicate that teachers’ effective use of technology to enhance their students’ learning opportunities in mathematics classrooms depends on the didactic possibilities linked with its use. Students’ exposure to SRL apparently enhances self-control, and self-monitoring of the cognitive processes that in turn affect algebraic thinking.

References

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Appendix

An algebraic activity with DERIVE software that enables users to substitute variables, simplify algebraic expressions, and solve equations.

<table>
<thead>
<tr>
<th>Algebraic activity</th>
<th>Enter option</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1: 0.4 ÷ 40</td>
<td>User</td>
</tr>
<tr>
<td>#2: 16</td>
<td>Simp (#1)</td>
</tr>
<tr>
<td>#3: 0.6x + 0.2x = 48</td>
<td>User</td>
</tr>
<tr>
<td>#4: ½x = 48</td>
<td>Solve (#3)</td>
</tr>
<tr>
<td>#5: x=60</td>
<td>Solve (#4)</td>
</tr>
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</table>