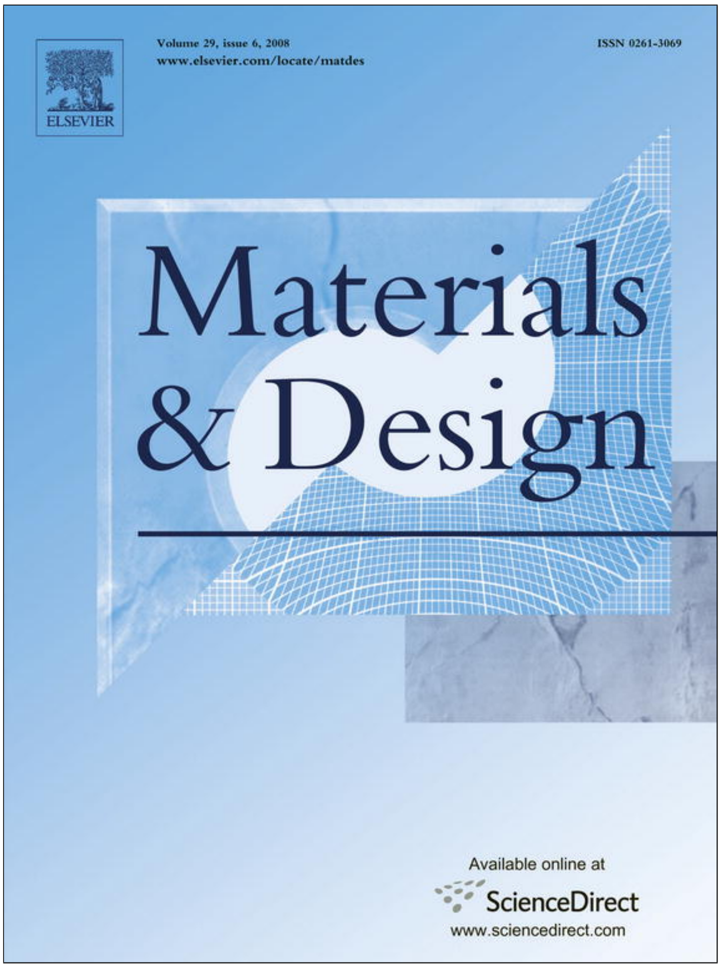

Volume 29, issue 6, 2008
www.elsevier.com/locate/matdes
ISSN 0261-3069
ELSEVIER
Materials & Design
Available online at
ScienceDirect
www.sciencedirect.com

Available online at www.sciencedirect.com
ScienceDirect
Materials and Design 29 (2008) 1170–1181
Materials & Design
www.elsevier.com/locate/matdes

# Statistical analysis of bending fatigue life data using Weibull distribution in glass-fiber reinforced polyester composites

Raif Sakin [a], İrfan Ay [b,*]

[a] Edremit Technical Vocational School of Higher Education, Balikesir University, Edremit, Turkey
[b] Department of Mechanical Engineering, Engineering and Architecture Faculty, Balikesir University, 10145 Balikesir, Turkey

Received 19 January 2007; accepted 16 May 2007
Available online 2 June 2007

**Abstract**

The bending fatigue behaviors were investigated in glass fiber-reinforced polyester composite plates, made from woven-roving with four different weights, 800, 500, 300, and 200 g/m$^2$, random distributed glass-mat with two different weights 225, and 450 g/m$^2$ and polyester resin. The plates which have fiber volume ratio $V_f \cong 44\%$ and obtained by using resin transfer moulding (RTM) method were cut down in directions of [0/90°] and [±45°]. Thus, eight different fiber-glass structures were obtained. These samples were tested in a computer aided fatigue apparatus which have fixed stress control and fatigue stress ratio [$R = -1$]. Two-parameter Weibull distribution function was used to analysis statistically the fatigue life results of composite samples. Weibull graphics were plotted for each sample using fatigue data. Then, *S–N* curves were drawn for different reliability levels ($R = 0.99$, $R = 0.50$, $R = 0.368$, $R = 0.10$) using these data. These *S–N* curves were introduced for the identification of the first failure time as reliability and safety limits for the benefit of designers. The probabilities of survival graphics were obtained for several stress and fatigue life levels. Besides, it was occurred that RTM conditions like fiber direction, resin permeability and full infiltration of fibers are very important when composites (GFRP) have been used for along time under dynamic loads by looking at test results in this study.


*Keywords:* Glass-fiber reinforced polyester composite (GFRP); Bending fatigue; Fatigue life; Weibull distribution; Mean life; Survival life; Reliability analysis

## 1. Introduction

In recent years, glass-fiber reinforced polyester (GFRP) composite materials have developed more rapidly than metals in structural applications. They are used alternatively instead of metallic materials because of their low density, high strength and rigidity [1–4]. The studies on reliability of structure depending on damage tolerance are very important for today's composite researchers since GFRP are used as preferable structures in fan blades, wind turbine, in air, sea and land transportation. Most of these materials are subjected to cycling loading during the service conditions. The mechanisms of composite materials under cycling loading and their fracture behaviors are really complex [1–4]. Because, anisotropy structure of GFRP materials forms three axial local stresses in itself. Static and fatigue failures in multilayer composites contain different damage combinations like matrix cracking, fiber–matrix debonding, ply delamination and fiber fracture. The form of each type failure is different depending on material properties, number of layers and loading type [3,5]. Thus, knowing the fatigue behavior under the cyclic loading is essential for using composite materials safely and in practical structural designs [1–4].

As it is in inhomogeneous all materials, it can be seen great differences in static strength and fatigue life test results among the samples in the same conditions in GFRP composites having anisotropy structure as well. Statistical evaluations are very important because of the different distribution of the test results in GFRP samples. When

* Corresponding author. Tel.: +90 266 612 1194; fax: +90 266 612 1257.
*E-mail address:* ay@balikesir.edu.tr (İ. Ay).

0261-3069/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.matdes.2007.05.005

great safety coefficient was used in the past, this distribution in the results was relatively unimportant. With the development of high performance aircraft, the changeability of mechanical properties of GFRP composites has gained great importance. Analyzing the reliability of composite materials is an inevitable need because of brittle fracture in structure and especially wide scatter of fatigue data. Thus, for the safe application of composite materials in industry, their fatigue data as statistically must be understood well. The statistical properties used, generally depend on usual distribution in mean strength. But especially, Weibull distribution has more reliable values than other distributions in fatigue data evaluations from the point of variables in life and strength parameters [3,6]. So, it has been proved in literature that Weibull distribution will be useful in the evaluation of fatigue data reliability in composite structures [3,6–10].

The aim of this study is to investigate the failure of fan blades and wind turbines made by composite materials instead of aluminum which is caused by bending fatigue forces. And then, the test results for these composites under reversal stress (bending) are to analysis as statistical by using Weibull distribution. Consequently, any engineer can use these *S–N* curves at the various reliability levels for their practical applications.

## 2. Experimental procedure

### 2.1. Materials and test specimens

General purposive and unsaturated polyester resin, glass-mat and woven-roving as shown in Table 1 were used for this study. In order to obtain GFRP sample by RTM method; heated mold system was constructed [11].

The mold was sprayed with a mold release agent to facilitate the later removal of the molding. Then one layer of glass-mat and one layer woven-roving put into the mold as shown in Fig. 1a and b. The properties of polyester resin other additive materials were prepared in accordance with RTM as shown in Table 1. Then, the prepared mixture was injected into the mold under pressure between 0.5–1 bar. At about 40 °C, 12 h later, the mold was opened and a plate with dimension of 310 × 600 × 3.00 mm was removed out of the mold. In the study, the volume of glass-fiber was obtained at $V_f \cong 44\%$ approximately. It was observed that bubbles were confined and incomplete infiltration occurred in some layers. Incomplete infiltrated plates were not tested and evaluated (see Fig. 2). Eight different material combinations were obtained by cutting the plates in the direction of [0/90°] and [±45] as shown in Table 2. Fatigue test samples were prepared from these plates with dimensions of 25 × 250 × 3.00 mm as shown in Fig. 3 [11]. By preparing samples with the dimension of 18 × 140 × 3.00 mm from the same plates according to the ASTM 790-00 three point bending tests were carried out and obtained maximum bending strength (Table 4) [12].

Especially in the formation of *S–N* curves, (Figs. 7 and 10) the maximum bending strength values were considered as one cycle strength value [11,13,14].

Table 1
Composition of GFRP plates

| | | |
|---|---|---|
| Matrix | Orthophthalic polyester resin[a] (Neoxil CE92N8) | |
| Monomer | Styrene[b] (15% of matrix volume) | |
| Catalyst | Cobalt naphthenate[b] (0.2% of matrix volume) | |
| Hardener | Methyl ethyl ketone peroxide[b] (MEKP) (0.7% of matrix volume) | |
| Reinforcement | *Woven-roving*[a,c] | |
| | Density: | 2.5 g/cm$^3$ |
| | Weight per unit area: | 800, 500, 300, and 200 g/m$^2$ |
| | Fiber direction: | [0/90°] and [±45°] |
| | *Glass-mat*[a,c] | |
| | Density: | 2.5 g/cm$^3$ |
| | Weight per unit area: | 225 and 450 g/m$^2$ |
| | Fiber direction: | Random |

[a] (CamElyaf A.S., Turkey).
[b] (Poliya A.S., Turkey).
[c] (Fibroteks A.S., Turkey).

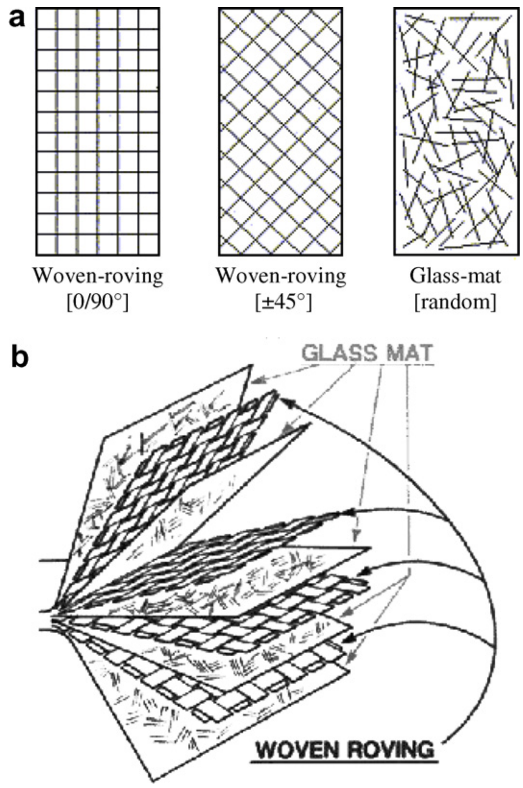



Fig. 1. (a) Schematic picture of glass-fiber used and (b) arrangement of glass-fiber.

## 3. Fatigue tests

Fatigue samples (shown in Fig. 3) were tested in the fatigue apparatus specially designed and improved by us (see Fig. 4) [11]. Test conditions were as follows:

Motor: 0.5 HP – 1390 rpm
Main shaft: 30 rpm (0.5 Hz)
Test frequency: 2 Hz
Temperature: room
Control: stress (load)
Loading ratio: $R = -1$
Maximal number of cycle: 1 million

The experimental conditions of the rotating fatigue tests were stress controlled flexural loading with a 30 rpm rotating speed. The maximum stress was applied on specimen

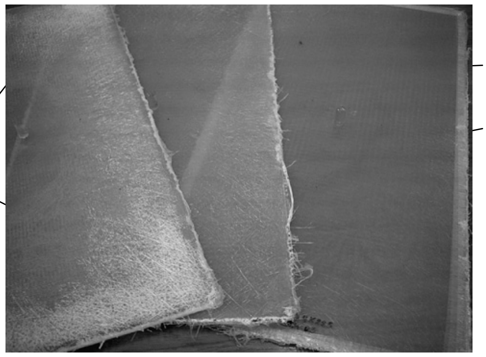



Fig. 2. Infiltration ratio in some laminates.

Table 2
GFRP sample groups and their structures ($V_f \cong 44\%$)

| Group | Fiber direction | Woven-roving | | | | Glass-mat | |
|---|---|---|---|---|---|---|---|
| | | 800 g/m$^2$ | 500 g/m$^2$ | 300 g/m$^2$ | 200 g/m$^2$ | 225 g/m$^2$ | 450 g/m$^2$ |
| A | [±45°] | 3[a] | – | – | – | 4 | – |
| B | [±45°] | – | 4 | – | – | 4 | 1 |
| C | [±45°] | – | – | 5 | – | 4 | 2 |
| D | [±45°] | – | – | – | 7 | 8 | – |
| E | [0/90°] | 3 | – | – | – | 4 | – |
| F | [0/90°] | – | 4 | – | – | 4 | 1 |
| G | [0/90°] | – | – | 5 | – | 4 | 2 |
| H | [0/90°] | – | – | – | 7 | 8 | – |

[a] Indicates number of layers in the sample.

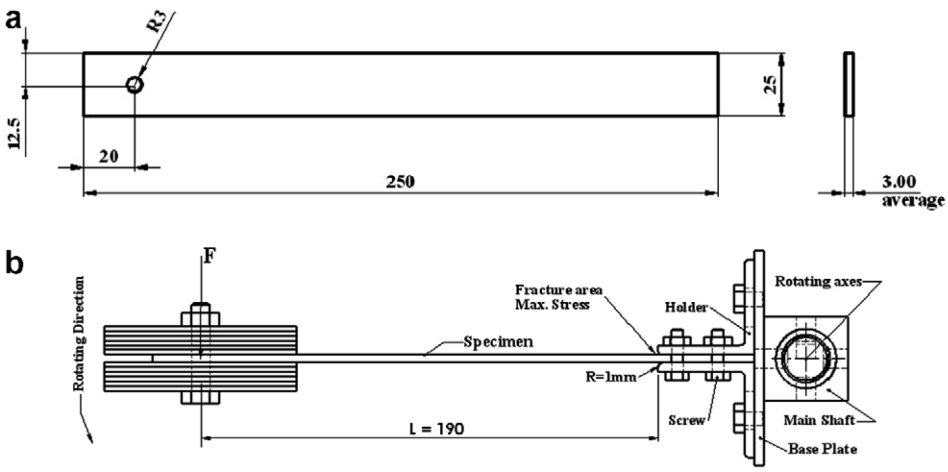



Fig. 3. (a) Fatigue sample dimensions and (b) sample connection position.

twice during one revolution (horizontal positions of a specimen), it might be logical to count two cycles for one revolution. Thus, the frequency of the rotating fatigue tests in this study was considered as 2 Hz [13]. The samples have been affected by lift and drag forces during the rotation in this fatigue test. Because of lower rotating speed, the effects of these forces were negligible. For loading the calculated weight to the specimens, steel disks, which had various dimensions and thicknesses (i.e. various weights), were used. Bending stresses have been only caused by the weights have been accepted as effective [11,13]. During the test, when the sample is horizontal position (0° and180°), maximum stress is occurred. The absolute values of these stresses are equal to each other ($\sigma_{max} = -\sigma_{min}$). During the rotation at 0° while the upper fibers are subjected to tension, the lower fiber are subjected to compression. When the sample position is 180°, upper fibers are subjected to compression and the lower fibers are subjected to tension. Thus, this stage was tension-compression fully reversed. In this situation fatigue stress ratio is $R = -1$.

The pictures of samples broken result of the fatigue test are shown in Fig. 5. Fiber-free bearings shows starting

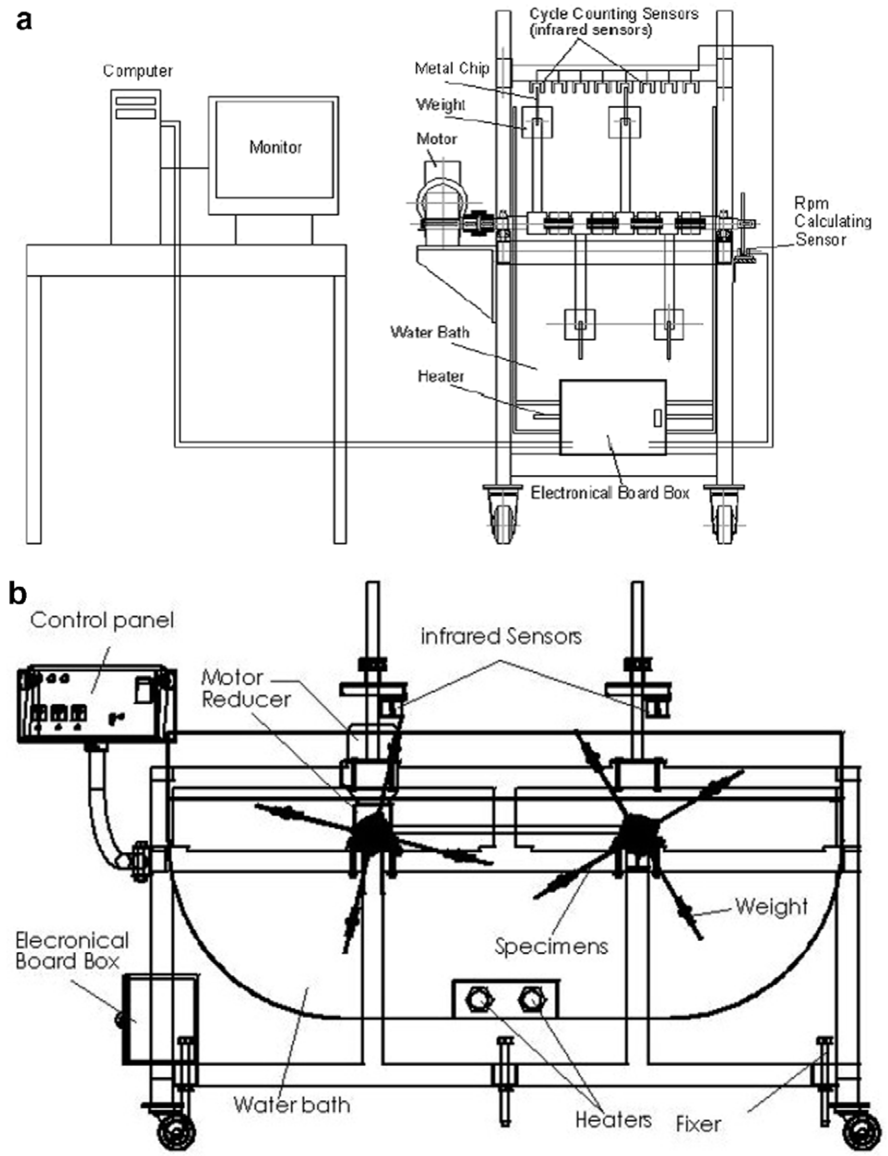



Fig. 4. (a) Front view and (b) left-side view. Schematic view of multi-specimen and fixed stress bending fatigue test apparatus.

point of the fatigue crack (Fig. 5b). This can also be considered as the weakest part of the sample. As the fatigue crack has occurred in the weakest part, matrix (polyester) and fiber (glass-fiber) separate from each other and the crack grows until the fiber breaks down. The weakest part in fiber–matrix interface is fiber-free bearings and dry fibers (not infiltrated) [15]. These samples have been not tested in this study. This condition is one of the most important points that should be taken into account. Transverse matrix- fiber separation generally starts at the upper fibers. The rigidity and flexibility of the upper and lower fibers decrease at the highest levels of the cycling bending process (tension-compression). Thus, the value of elasticity module will reduce by passage of time under cycling loading [16]. On the other hand, the separation between longitudinal fiber and matrix is similar to the separation in the transverse matrix cracks [16].

## 4. Statical analysis of fatigue life data

### 4.1. Theory of Weibull distribution

Weibull distribution is being used to model extreme values such as failure times and fatigue life. Two popular forms of this distribution are two- and three-parameter Weibull distributions. The probability density function (PDF) of two-parameter distribution has been indicated in the following Eq. (1). This PDF Equation is the most known definition of two-parameter Weibull distribution [2,3,17,18]

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \quad \alpha \geqslant 0, \;\; \beta \geqslant 0 \tag{1}$$

where $\alpha$ and $\beta$ is the scale, shape parameter. The advantages of two-parameter Weibull distribution are as follows [3]:

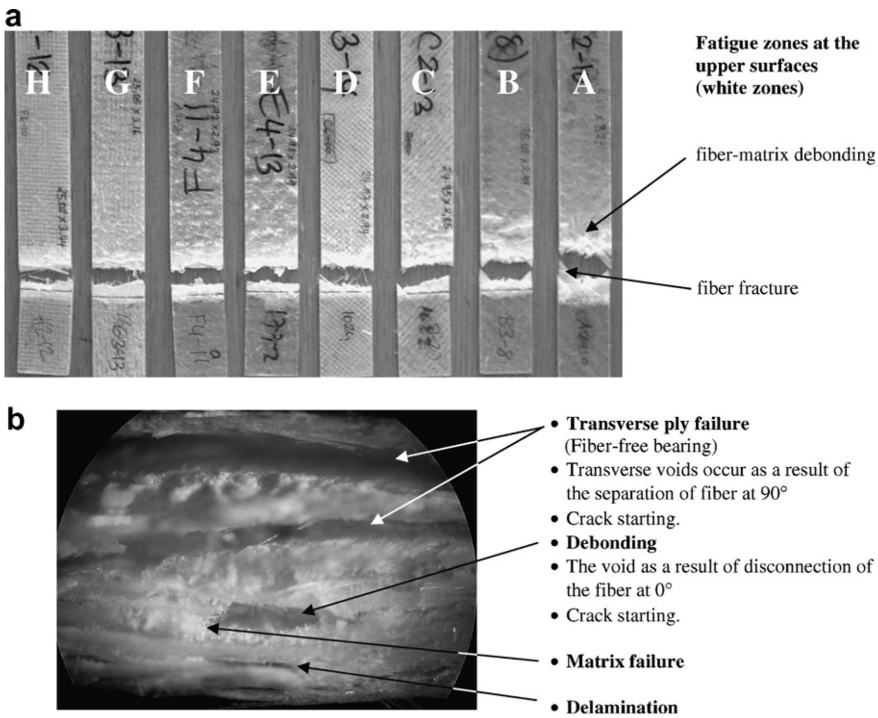



Fig. 5. (a) The pictures of the samples broken as a result of the fatigue tests and (b) front view of cracked zone as a result of fatigue (Group: E, zoom: 40X). Pictures of GFRP samples.

- It can be explain with a simple function and applied easily.
- It is used frequently in the evaluation of fatigue life of composites.
- Its usage is easy having present graphics and simple calculation methods.
- It gives some physical rules concerning failure when the slope of the Weibull probability plots taken into account.

If PDF Equation is integrated, cumulative density function (CDF) in Eq. (2) is obtained. Eq. (3) derives from Eq. (2).

$$F_f(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \tag{2}$$

$$1 - F_f(x) = e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \tag{3}$$

$$F_s(x) = 1 - F_f(x) \tag{4}$$

$$R_x = 1 - P_x \tag{5}$$

In the above equations;

- $x$ variable (usually life). Failure cycles in this study ($N_f$),
- $\beta$ shape parameter or Weibull slope,
- $\alpha$ characteristic life or scale parameter,
- $F_f(x)$ probability of failure ($P_x$),
- $F_s(x)$ probability of survival or reliability ($R_x$).

If the natural logarithm of both sides of the Eq. (3) is taken, the following Eq. (6) can be written.

$$\ln\left(\ln\left(\frac{1}{1 - F_f(x)}\right)\right) = \beta \ln(x) - \beta \ln(\alpha) \tag{6}$$

When the Eq. (6) is rearranged as linear equation, $Y = \ln(\ln(1/(1 - F_f(x))))$, $X = \ln(x)$, $m = \beta$ and $c = -\beta(\ln(\alpha))$ is written. Hence, a linear regression model in the form of Eq. (7) is obtained

$$Y = mX + c \tag{7}$$

$$\alpha = e^{(-c/\beta)} \tag{8}$$

In Eq. (2), when $x = \alpha$,

$$F_f(x) = 1 - e^{-(1)^{\beta}}$$

$$F_f(x) = 1 - 0.368$$

$F_f(x) = P_x = 0.632 = 63.2\%$ is obtained.

According to Eq. (8) characteristic life ($\alpha$) is the time or the number of cycles at which 63.2% of the population is expected to fail. The life of critical parts (roller bearing, blade etc.) designed for fatigue is indicated as $P_{10}$, $P_1$, $P_{0.1}$ for lower failure probabilities [19]. In this study, as shown in Fig. 10, $S$–$N$ plots were drawn for the values of $P_1$, $P_{50}$, $P_{63.2}$, $P_{90}$ (or $R_{99}$, $R_{50}$, $R_{36.8}$, $R_{10}$) and this study also guides the designers. $N_{P_x}$ or $N_{R_x}$ are values of life indicating $X$% failure probability and can be calculated from Eq. (9). The median life values (50% life) can be calculated Eq. (10) or can be read from the graphics in Fig. 11. In this study, survival graphics drawn for each stress value of GFRP samples is given in Fig. 11.

$$N_{P_x} = N_{R_x} = \alpha \cdot ((-\ln(R_x))^{1/\beta}) \tag{9}$$

$$N_{P_{50}} = N_{R_{50}} = \alpha \cdot ((-\ln(2))^{1/\beta}) \tag{10}$$

Mean life (mean time to failure = MTTF = $N_0$), standard deviation (SD) and coefficient of variation (CV) of two-parameter Weibull distribution were calculated from the following Equations [3,18,20,21]

$$\mathrm{MTTF} = N_0 = \alpha \cdot \Gamma(1 + 1/\beta) \quad (11)$$

$$\mathrm{SD} = \alpha \cdot \sqrt{\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)} \quad (12)$$

$$\mathrm{CV} = \frac{\mathrm{SD}}{N_0} = \frac{\alpha \cdot \sqrt{\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)}}{\alpha \cdot \Gamma(1 + 1/\beta)} \quad (13)$$

where ($\Gamma$) is gamma function.

### 4.2. Application of Weibull distribution

The drawing of Weibull line for $X$ and $Y$, the parameter of Weibull distribution and reliability analysis processes can be carried out by software such as Microsoft Excel and SPSS [17,22]. Microsoft Excel has been used in this study. The following processes were carried out to draw Weibull lines and obtain parameters.

1. The number of failure cycle corresponding to each stress was located successively.
2. Serial number was given for each value ($i = 1, 2, 3, \ldots, n$).
3. Each value for failure probability was used in Bernard's Median Rank formula given in Eq. (14).

$$\mathrm{MR} = \frac{i - 0.3}{n + 0.4} \quad (14)$$

where $i$ is failure serial number and $n$ is total test number of samples [22–24].

4. $\ln(\ln(1/(1 - \mathrm{MR})))$ values were calculated for each cycle value ($Y$-axis).
5. ln(cycle) values were calculated for each cycle value ($X$-axis).
6. Only the data given for group-A samples as example in Table 3 was transferred to Microsoft Excel. For regression analysis, the Analysis ToolPak Add-In was loaded into Microsoft Excel [11,22].
7. The graphics of ln(cycle) and $\ln(\ln(1/(1 - \mathrm{MR})))$ values were drawn as shown in Fig. 6.
8. $Y = mX + c$ linear equation given in the Eq. (7) was obtained in the most reasonable form from this graphics.

Table 3
Summarized Weibull values of GFRP samples for Group-A [11]

| Stress amplitude, $S_a$ (MPa) | Cycle | Rank | Med. Rnk. MR | ln(cycle) ($X$-axis) | $\ln(\ln(1/(1 - \mathrm{MR})))$ ($Y$-axis) | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|---|---|
| | 360 | 1 | 0.129630 | 5.886104 | −1.974459 | | |
| | 736 | 2 | 0.314815 | 6.601230 | −0.972686 | | |
| 106.728 | 922 | 3 | 0.500000 | 6.826545 | −0.366513 | 947 | 2.204 |
| | 1010 | 4 | 0.685185 | 6.917706 | 0.144767 | | |
| | 1016 | 5 | 0.870370 | 6.923629 | 0.714455 | | |
| . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . |
| | 960,633 | 1 | 0.129630 | 13.775348 | −1.974459 | | |
| | 979,766 | 2 | 0.314815 | 13.795069 | −0.972686 | | |
| 50.257 | 1,088,925 | 3 | 0.500000 | 13.900702 | −0.366513 | 1,103,609 | 11.800 |
| | 1,108,771 | 4 | 0.685185 | 13.918763 | 0.144767 | | |
| | 1,170,104 | 5 | 0.870370 | 13.972603 | 0.714455 | | |

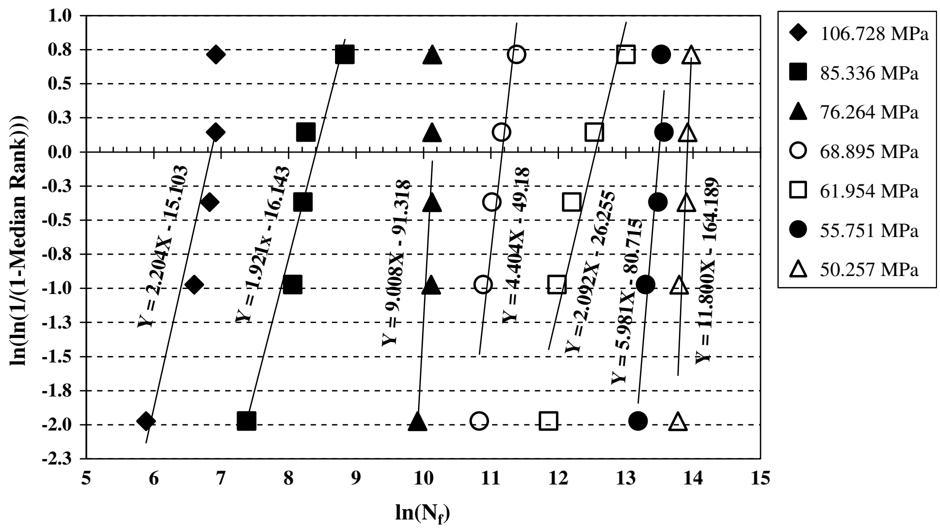



Fig. 6. Weibull lines for GFRP samples in Group-A.

Table 4
According to test results, Weibull parameters for each stress amplitude [11]

| Stress amplitude, $S_a$ (MPa) | Characteristic life, ($\alpha$) (cycle) | Shape parameter ($\beta$) | Weibull mean life (cycle) |
|---|---|---|---|
| Group: A | | | |
| 203.120 | 1 | 1.000 | 1 |
| 106.728 | 947 | 2.204 | 839 |
| 85.336 | 4472 | 1.921 | 3967 |
| 76.268 | 25,270 | 9.008 | 23,930 |
| 68.895 | 70,756 | 4.404 | 64,490 |
| 61.954 | 282,538 | 2.092 | 250,249 |
| 55.751 | 725,345 | 5.981 | 672,801 |
| 50.257 | 1103 609 | 11.800 | 1,056,907 |
| Group: B | | | |
| 258.288 | 1 | 1.000 | 1 |
| 116.764 | 917 | 5.112 | 843 |
| 93.788 | 8204 | 4.015 | 7438 |
| 84.403 | 60,613 | 6.229 | 56,348 |
| 76.105 | 186,720 | 2.730 | 166,111 |
| 68.653 | 468,909 | 4.552 | 428,201 |
| 61.550 | 779,735 | 5.091 | 716,678 |
| 57.922 | 1,253,868 | 4.313 | 1,141,429 |
| Group: C | | | |
| 278.313 | 1 | 1.000 | 1 |
| 128.039 | 1422 | 3.597 | 1281 |
| 103.769 | 8184 | 2.533 | 7264 |
| 93.235 | 21,058 | 3.766 | 19,022 |
| 84.152 | 64,604 | 3.797 | 58,386 |
| 75.485 | 162,344 | 4.628 | 148,392 |
| 68.028 | 351,042 | 2.773 | 312,474 |
| 63.378 | 915,776 | 10.639 | 873,466 |
| 61.221 | 1,262,614 | 9.156 | 1,196,578 |
| Group: D | | | |
| 265.468 | 1 | 1.000 | 1 |
| 111.502 | 1182 | 1.719 | 1053 |
| 97.072 | 4125 | 1.678 | 3684 |
| 87.219 | 17,128 | 2.825 | 15,257 |
| 78.800 | 44,060 | 3.066 | 39,383 |
| 70.792 | 192,165 | 1.830 | 170,759 |
| 63.757 | 459,053 | 2.674 | 408,096 |
| 57.411 | 807,018 | 22.523 | 787,848 |
| 53.070 | 1,556,050 | 4.324 | 1,416,720 |
| Group: E | | | |
| 353.540 | 1 | 1.000 | 1 |
| 163.271 | 1233 | 1.864 | 1095 |
| 129.452 | 5971 | 3.988 | 5412 |
| 106.908 | 49,840 | 4.088 | 45,231 |
| 96.166 | 269,491 | 2.308 | 238,755 |
| 91.170 | 617,135 | 10.157 | 587,497 |
| 88.734 | 887,796 | 6.363 | 826,281 |
| 86.640 | 1,109,131 | 14.472 | 1,069,822 |
| 84.250 | 1,532,755 | 4.910 | 1,405,857 |
| Group: F | | | |
| 375.200 | 1 | 1.000 | 1 |
| 151.934 | 3195 | 2.532 | 2836 |
| 123.852 | 9534 | 2.047 | 8446 |
| 111.613 | 39,695 | 2.388 | 35,185 |
| 100.138 | 216,224 | 4.479 | 197,266 |
| 90.322 | 349,306 | 6.519 | 325,527 |
| 81.316 | 670,939 | 5.040 | 616,318 |
| 73.201 | 1,053,763 | 10.309 | 1,003,780 |

Table 4 (*continued*)

| Stress amplitude, $S_a$ (MPa) | Characteristic life, ($\alpha$) (cycle) | Shape parameter ($\beta$) | Weibull mean life (cycle) |
|---|---|---|---|
| Group: G | | | |
| 348.198 | 1 | 1.000 | 1 |
| 147.771 | 1354 | 1.351 | 1241 |
| 119.308 | 16,786 | 3.177 | 15,029 |
| 107.214 | 51,365 | 2.322 | 45,510 |
| 96.241 | 139,934 | 2.227 | 123,935 |
| 86.771 | 254,726 | 8.218 | 240,198 |
| 77.882 | 481,137 | 7.360 | 451,235 |
| 73.941 | 1,244,927 | 4.278 | 1,132,769 |
| Group: H | | | |
| 311.504 | 1 | 1.000 | 1 |
| 145.837 | 2011 | 1.388 | 1835 |
| 117.274 | 9379 | 3.211 | 8402 |
| 105.754 | 41,617 | 25.650 | 40,741 |
| 95.132 | 77,664 | 4.330 | 70,716 |
| 85.619 | 175,290 | 3.669 | 158,117 |
| 76.938 | 384,464 | 4.135 | 349,141 |
| 69.377 | 785,661 | 9.065 | 744,236 |
| 64.270 | 1,595,150 | 6.289 | 1,483,682 |

9. $\beta$ and $c$ values were obtained by linear regression application (least squares method). $m = \beta$ parameter was obtained directly from the slope of the line.
10. $\alpha$ parameter was obtained from the Eq. (8)
11. The mean fatigue life corresponding to each stress was calculated from Eq. (11), and the variation coefficients were calculated from Eq. (13). The difference between mean fatigue life and the variation coefficients were given in Fig. 9.
12. The above processes were carried out in order for all samples group and Weibull graphics and parameters were obtained $\alpha$ and $\beta$ parameters obtained are shown in Table 4.

The results of the processes carried out above have been summarized in Table 3. Example Weibull graphics for each stress value has been given in Fig. 6 [11].

## 5. Results and discussion

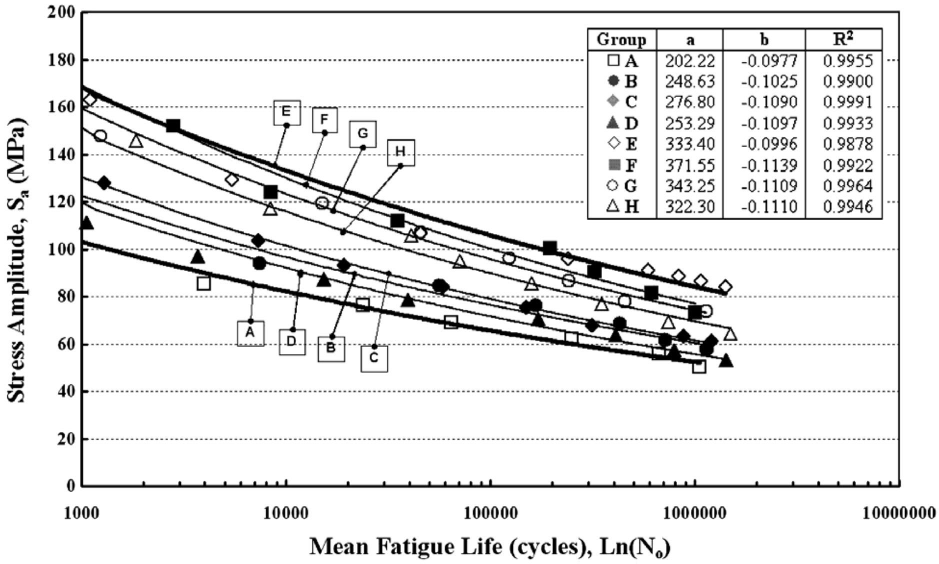
### 5.1. The S–N curves

$10^6$ cycles which is corresponding to fatigue strength has been taken into account as a failure criterion in the evaluation of fatigue tests [11,13,25]. The *S–N* curves obtained for eight different average fatigue life of GFRP samples have been shown in Fig. 7. Power Function has been used in Eq. (15) for the evaluation of fatigue test data [2,3,6,11,26].

$$S_a = a \cdot (N_f)^b \tag{15}$$

In this equation;

$S_a$ stress amplitude (fatigue strength),




($R^2$ = correlation coefficient (0 to 1). The correlation coefficients indicates the Weibull is a good fit)

Fig. 7. *S–N* curves for eight different samples.

$N_f$ number of cycle (fatigue life),
*a* and *b* are constants (It's given for each material group in Fig. 7).

Table 5
The stress (fatigue strength) values and decrease rate in $10^6$ cycles

| Group | Stress amplitude, $S_a$ (MPa) | Decrease rate (%) |
|---|---|---|
| E | 84.210 | 37.7 |
| A | 52.435 | |
| F | 77.022 | 21.7 |
| B | 60.333 | |
| G | 74.167 | 17.2 |
| C | 61.400 | |
| H | 69.544 | 20.0 |
| D | 55.644 | |

The effects on fatigue strength are obtained from *S–N* curves for each GFRP group. They are fiber direction, weight per unit area, maximum stress values corresponding to $10^6$ cycles and decrease rate in maximum stress amplitude values of fibers having same weight per unit area but in different directions. These effects have been shown in Table 5 and Fig. 8.

We can make the following comment for the directions of [0/90°], [±45°] in woven-roving composites which have 800, 500, 300 and 200 g/m$^2$ weight per unit area and having the same volume ($V_f \cong 44\%$) from Table 5 and Fig. 8.

The change in fatigue strength corresponding to $10^6$ cycles depends on fiber direction and weight per unit area can be seen in Table 5 and Fig. 8. The strength in the direction of [0/90°] is higher. On the other hand, the strength

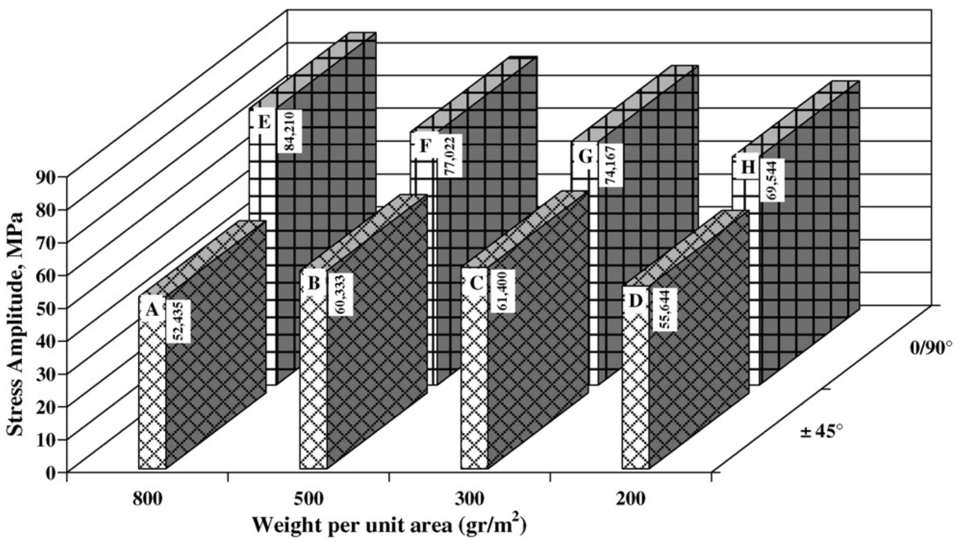



Fig. 8. The relationship of stress amplitude, weight per unit area and fiber direction.

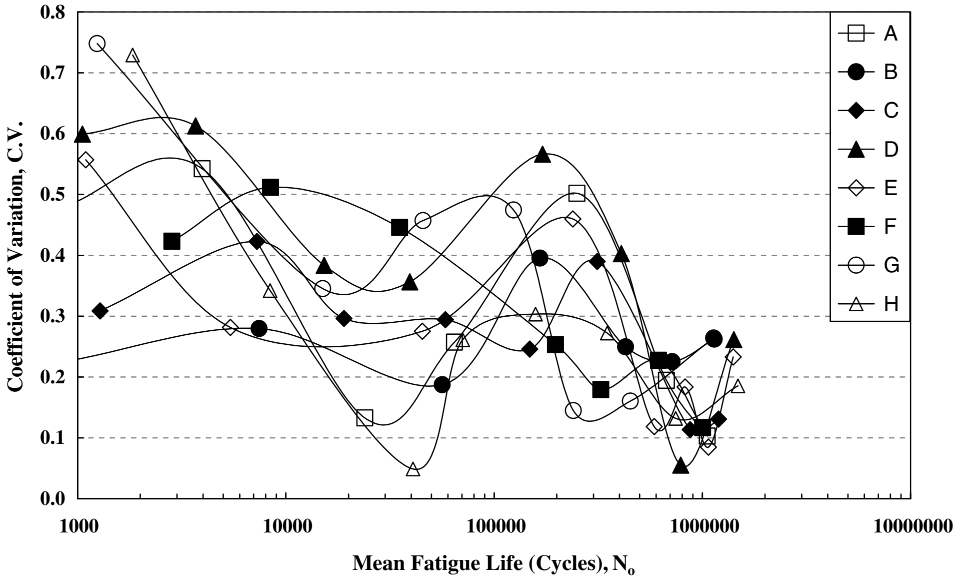



Fig. 9. Effect of mean fatigue life on the coefficient of variation, CV.

has decreased suddenly because of existing of the shear stresses formed in weak interfaces on planes at [±45°] and fibers at the direction of [±45°] have had to carry over load nearly 1.9 times more than that of the fibers at the direction of [0/90°]. Thus, anisotropy property between the samples of groups E&A is dominant reduced fatigue strength at the rate of 37.7%. Because of high resin permeability of composite having groups E&A woven-roving, infiltrations is better but the weak matrix cross-section is bigger than that of composite having groups F&B, G&C and H&D [11,27].

### 5.2. Scatter in the fatigue life results

CV values for fatigue life of GFRP samples have calculated by using Eq. (13). Coefficient of variation (CV) graphics which is calculated by Eq. (11) corresponding to mean life (MTTF) has been shown in Fig. 9. According to these results, scatter in the fatigue life values has the widest between $10^3$–$10^4$ and $10^5$–$10^6$. The first widest scatter was observed at the life range $10^3$–$10^4$ cycles for GFRP samples because of the larger defects in structure at high stress level at the beginning of test. But later, the second widest scatter was observed at the life range $10^5$–$10^6$ cycles. Because, the small defects in structure reach a critical value at the different stress level. This trend for different sample group is extremely important for the application and design of GFRP structure [3].

### 5.3. Reliability analysis of fatigue results and bounds for the S–N curves

The term Reliability is used for the probability of functional performance of a part under current service condition and in definite time period. This also is known as the probability of survival [3,28]. The probability of survival graphics corresponding to each stress values of GFRP samples has been shown in Fig. 11. These graphics have been drawn by using Eqs. (3) and (4). The probability 50% survival of samples from these diagrams is intersected by drawing horizontal line from *Y*-axis, hence the probability can be found in which cycle value it has. For example as shown in the diagram in Fig. 11, while the stress is 50.257 MPa and failure cycles are 1069858 for group-A samples, these values are 86.640 MPa and 1081394 cycles for group-E samples corresponding to 50% Reliability.

The bending fatigue test results of GFRP materials have been scattered in a great scale because of their anisotropy structures and semi-brittle behaviours. Safe life = Reliability is an important parameter for design in this type of structure. Reliability means that "a material can be used without failure". The definition of reliability in engineering has been shown in Figs. 10 and 11. Fig. 10 shows *S–N* curves belong to several reliability levels of GFRP samples. As *S–N* curves belong to average values for each sample in Fig. 7 and the curves having 50% reliability in Fig. 10 are closer to each other, the *S–N* curve obtained from the average fatigue data in scattered position can be accepted as 50% reliability of survival as well. The *S–N* curves have been given belonging to four different safe levels ($R = 0.99$, $R = 0.50$, $R = 0.368$, $R = 0.10$) in order, in Fig. 10. These *S–N* curves provide possibility of prediction reliability fatigue life needed to designer.

## 6. Conclusions

In this study, the following results have been obtained.

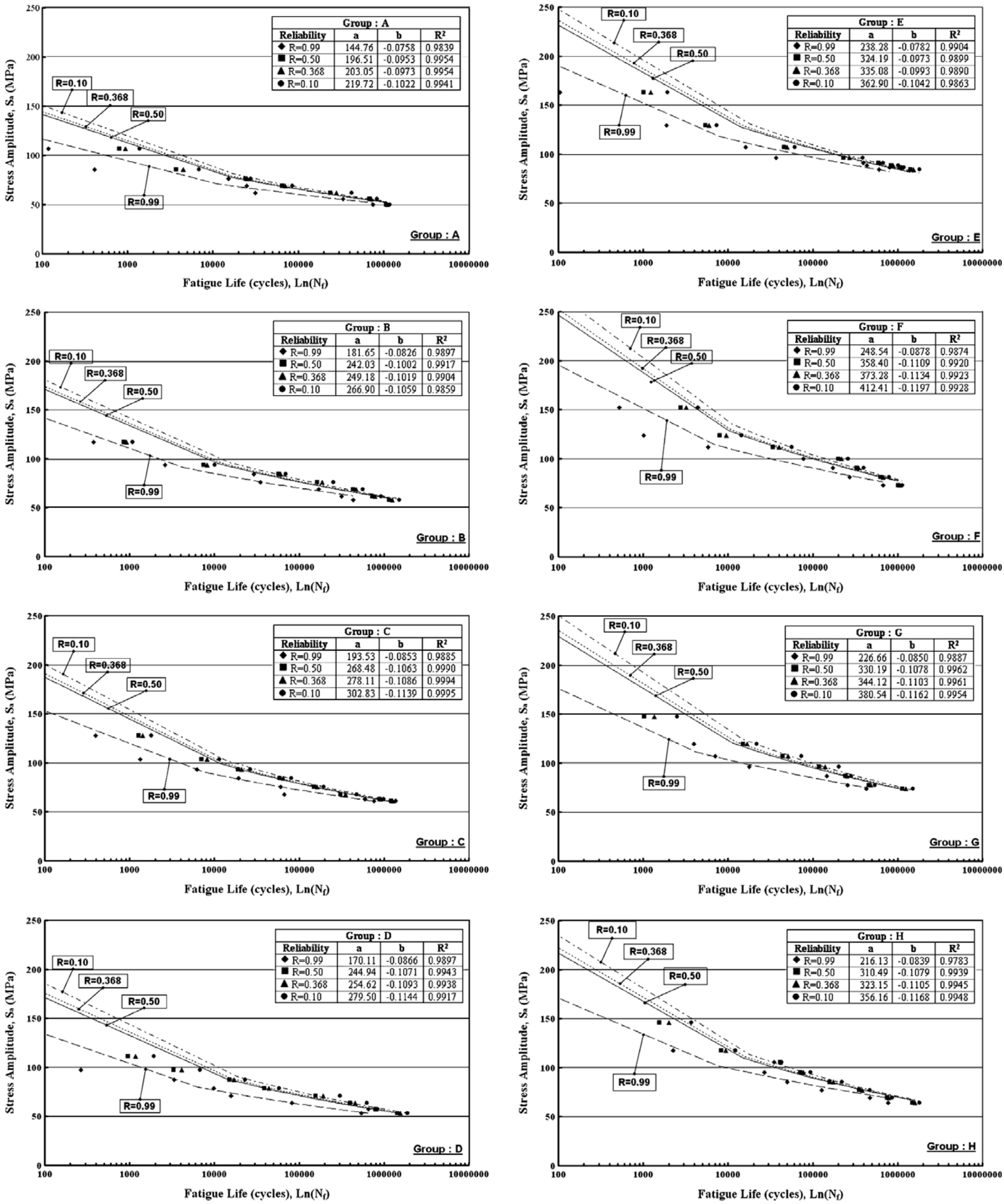



Fig. 10. The *S–N* curves for different reliable levels (A–B–C–D–E–F–G–H).

(1) It has been observed that the lowest fatigue strength is in group-A, the highest fatigue strength is in group-E.

(2) The *S–N* curves depending on mean life values for all groups of GFRP composites have been presented in order practicing engineers could find them useful.

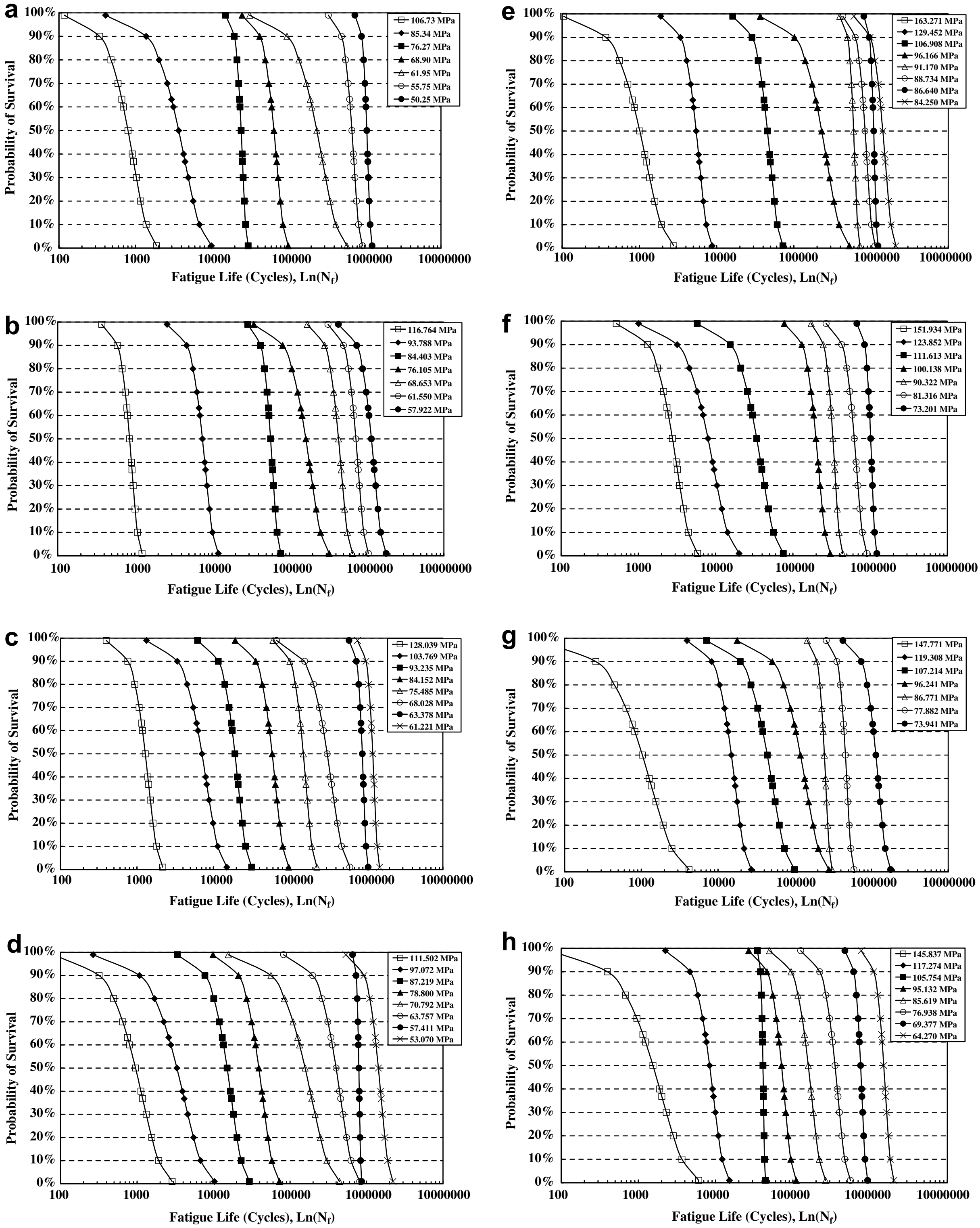



Fig. 11. Probability of survival graphs for GFRP specimens.

(3) The most interesting result is the fatigue strength difference (37.7%) between the samples having the same fiber weight in group E&A. Generally, this sudden strength reduction resulting from the change of fiber direction, in GFRP samples is a very important point that should be taken into account in designs [11].

(4) As group E&A specimens have more resin permeability, fatigue data distribution of these samples have less scattering because of full infiltration during RTM. The group H&D specimens having less resin permeability and incomplete infiltration have the widest scatter in life values under the high stress.

Also, they showed sudden fracture behaviors. This criteria is extremely important in high stress and low cycle processes.

(5) The scatter value in all sample groups has been decreased in about $10^6$ cycles taken as a failure criterion and the scatter values have been closer to each other. This result is useful for designers because of the accuracy and exact repeatability of the test in about $10^6$ cycles.

(6) Safe design life for brittle structured composites has great importance. *S–N* curves for reliability levels as $R = 0.99$, $R = 0.50$, $R = 0.368$, and $R = 0.10$ have been drawn and presented for designers. These diagrams can be considered as reliability or safety limits in identification of the first failure time of a component under any stress amplitude. Especially, the usage of *S–N* curves ($R = 0.99$) should be advised in the design of air-craft which have to have higher safety and reliability.

(7) Fatigue life distribution diagrams have been obtained by using two-parameter Weibull distribution function for GFRP composites. The reliability percentage (%) can be found easily corresponding to any life (cycle) or stress amplitude from these diagrams.

## Acknowledgements

The study has been partly granted by Unit of Scientific Research Projects in Balıkesir University. Besides, the authors thank Fibroteks Woven Co. and Glass-Fiber Co. (Şişecam) for their material and workmanship support.

## References

[1] Tomita Y, Morioka K, Iwasa M. Bending fatigue of long carbon fiber-reinforced epoxy composites. Mater Sci Eng A 2001;319–321:679–82.

[2] Abdallah MH, Abdin Enayat M, Selmy AI, Khashaba U. Short communication effect of mean stress on fatigue behavior of GFRP pultruded rod composites. Compos Part A: Appl Sci Manuf 1997;28(1):87–91.

[3] Khashaba UA. Fatigue and reliability analysis of unidirectional GFRP composites under rotating bending loads. J Compos Mater 2003;37(4):317–31.

[4] Van Paepegem W, Degrieck J. Experimental set-up for and numerical modeling of bending fatigue experiments on plain woven glass/epoxy composites. Compos Struct 2001;51(1):1–8.

[5] Amateau MF. Engineering composite materials (EMCH471) course. The Pennsylvania State University; 2003.

[6] Abdallah MH, Abdin EM, Selmy AI, Khashaba UA. Reliability analysis of GFRP pultruded composite rods. Int J Qual Reliab Manage 1996;13(2):88–98.

[7] J Lee B, Harris DP, Almond, Hammett F. Fibre composite fatigue-life determination. Compos Part A: Appl Sci Manuf 1997;28(1):5–15.

[8] Lin SH, Ma CCM, Tai NH. Long fiber reinforced polyamide and polycarbonate composites. II: Fatigue behavior and morphological property. J Vinyl Addit Technol 2004;2(1):80–6.

[9] Pizhong Qiao, Mijia Yang. Fatigue life prediction of pultruded E-glass/polyurethane composites. J Compos Mater 2006;40(9):815–37.

[10] Harris B. A parametric constant-life model for prediction of the fatigue lives of fibre-reinforced plastics. Fatigue in Composites. Boca Raton, Florida, USA, Abington, UK: Woodhead Publishing, CRC Press; 2003 [pp. 546–68].

[11] Sakin R. The design of test machine with computer aided and multi-specimen for bending fatigue and the investigation of bending fatigue of glass-fiber reinforced polyester composites. Ph.D. Thesis, University of Balikesir, Turkey; 2004.

[12] Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials, ASTM D790-00; 2001.

[13] Kim HY, Marrero TR, Yasuda HK, Pringle OA. A simple multi-specimen apparatus for fixed stress fatigue testing. J Biomed Mater Res 1999;48(3):297–300.

[14] O'Brien TK, Chawan AD, Krueger R, Paris IL. Transverse tension fatigue life characterization through flexure testing of composite materials. Int J Fatigue 2002;24(2):127–45.

[15] Degallaix G, Hassaini D, Vittecoq. Cyclic shearing behavior of a unidirectional glass/epoxy composite. Int J Fatigue 2002;24(2–4):319–26.

[16] Liao K, Schultheisz CR, Hunston DL. Long-term environmental fatigue of pultruded glass-fiber-reinforced composites under flexural fatigue. Int J Fatigue 1999;21(5):485–95.

[17] Dirikoğlu MH, Aktaş A. Statistical analysis of fracture strength of composite materials using Weibull distribution. Turkish J Eng Environ Sci 2002;26:45–8.

[18] Zhou G, Davies GAO. Characterization of thick glass woven-roving/polyester laminates: 2. Flexure and statistical considerations. Composites 1995;26(8):587–96.

[19] Jess Comer (Professor, Dr.). Iowa State University Mechanical Engineering (ME), ME Courses, Advanced Machine Design II (ME-515), Lecture 5; 2000.

[20] Paul Tobias. Engineering Statistics Handbook, Assessing Product Reliability, Introduction, What are the basic lifetime distribution models used for non-repairable populations? Weibull; 2006.

[21] Paul Barringer. Reliability engineering consulting and training. Barringer and Associates, Inc; 2000.

[22] William W Dorner. Using Excel for Data Analysis, Using Microsoft Excel for Weibull Analysis; 1999.

[23] ReliaSoft Corporation, The Weibull Distribution; 2006.

[24] Tai NH, Ma CCM, Lin JM, Wu GY. Effects of thickness on the fatigue-behavior of quasi-isotropic carbon/epoxy composites before and after low energy impacts. Compos Sci Technol 1999;59(11):1753–62.

[25] Ben Zineb T, Sedrakian A, Billoet JL. An original pure bending device with large displacements and rotations for static and fatigue tests of composite structures. Compos Part B: Eng 2003;34(5):447–58.

[26] Tai N-H, Yip M-C, Tseng C-M. Influences of thermal cycling and low-energy impact on the fatigue behavior of carbon/PEEK laminates. Compos Part B: Eng 1999;30(8):849–65.

[27] Sözer M. Control of Composite Material Production by RTM aided with Simulations. Tübitak-Misag Project, Project No. 192; 2003.

[28] Akkurt M. The fundamentals of reliability at Mechanical Construction. Chamber of Mechanical Engineer, No. 106, Istanbul; 1977.